Journal
of
The Franklin Institute

EDITOR, HENRY BUTLER ALLEN, MET.E., D.SC., SC.D., D.ENG.
ASSISTANT EDITOR, NANCY S. GLENN, M.A.

Associate Editors:

WILDER D. BANCROFT, PH.D.
CHARLES B. BAZZONI, PH.D.
ARTHUR L. DAY, SC.D.
R. EKSERGIAN, PH.D.
PAUL D. FOOTE, PH.D.
C. E. K. MEES, D.SC.
WILLIAM B. MELDRUM, PH.D.
HENRY C. SHERMAN, SC.D.
W. F. G. SWANN, D.SC.
HUGH S. TAYLOR, D.SC.
A. F. ZAHM, PH.D.
JOHN ZELENY, PH.D.

Committee on Publications:

CHARLES B. BAZZONI, CHAIRMAN
G. H. CLAMER
RICHARD HOWSON
CLARENCE L. JORDAN
LIONEL F. LEVY
HIRAM S. LUKENS
WINTHROP R. WRIGHT

Vol. 249 **MARCH, 1950** **No. 3**

CONTENTS

Henry Cavendish, Electrician . . . . CHARLES SUSSKIND 181
A Note on the Solution of Certain Approximation Problems in Network Analysis . . . . R. M. FANO 189
"Developable" and Developed Silver Halide Grains . . . . LUDWIK SILBERSTEIN 207
The Part of Octahedral Theory in the Study of the Plasticity of Metals . . . . WENDELL P. ROOP 223
Notes from the National Bureau of Standards 237
The Franklin Institute
    Minutes of Stated Meeting 241
    Journal of The Franklin Institute 241
    Museum 242
    The Franklin Institute Laboratories 243
    Library 245
    Membership 247
Notes from the Biochemical Research Foundation 248
Book Reviews 252
Book Notes 260
Current Topics 188, 206, 262

Published at
Prince and Lemon Streets, Lancaster, Penna., by
THE FRANKLIN INSTITUTE OF THE STATE OF PENNSYLVANIA
Benjamin Franklin Parkway at Twentieth St., Philadelphia 3, Penna.

DOMESTIC—EIGHT DOLLARS PER YEAR **FOREIGN—NINE DOLLARS PER YEAR**
(Foreign Postage Additional)
SINGLE CURRENT NUMBERS—ONE DOLLAR EACH
REPRINTS—FIFTY CENTS PER INDIVIDUAL INSTALLMENT

Indexes to the semi-annual volumes of the JOURNAL
are published with the June and December numbers

[illegible]

[illegible]

FIDELITY-PHILADELPHIA
Trust Company

[illegible]

Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 249 MARCH, 1950 No. 3

HENRY CAVENDISH, ELECTRICIAN

BY

CHARLES SUSSKIND[1]

The name of the discoverer of the composition of water is primarily associated with chemistry. Yet the accounts of the electrical researches of Henry Cavendish, found after his death among his manuscripts, show him to have been one of the foremost physicists of all times as well. He published only two of his papers on electricity; but the work described in the remaining manuscripts anticipated some of the contributions of Ohm and Coulomb. So far ahead of his contemporaries was he that the great Maxwell himself, who spent the last five years of his life editing Cavendish's papers on electricity, wrote in 1874: "His measures of capacity will give us some work . . . before we work up to the point where he left it." A century had elapsed since Cavendish had made those "measures of capacity," but science had not yet caught up with him!

Henry Cavendish (1731–1810) belonged to one of the most aristocratic families in England: both his grandfathers were dukes. In view of this exalted lineage, it is amazing how little is known of his private life. We know that he spent nearly four years at Cambridge, but left without taking a degree. During his father's lifetime he lived in very modest circumstances. It was during this period that he probably acquired those traits of character which marked him forever afterwards as a retiring, taciturn, almost morbidly shy man. A contemporary, Lord Brougham, remarked of Cavendish: "He probably uttered fewer words in the course of his life than any man who ever lived to fourscore years, not at all excepting the monks of La Trappe." His father's death, together with the legacy from an uncle, served to turn him into a fabulously rich man. He became a millionaire, a condition as rare among the scientists of his time as it is today, and one which caused the French biographer Biot to call Cavendish epigrammatically *le plus riche de tous les savants, et probablement aussi, le plus savant de tous les riches.*

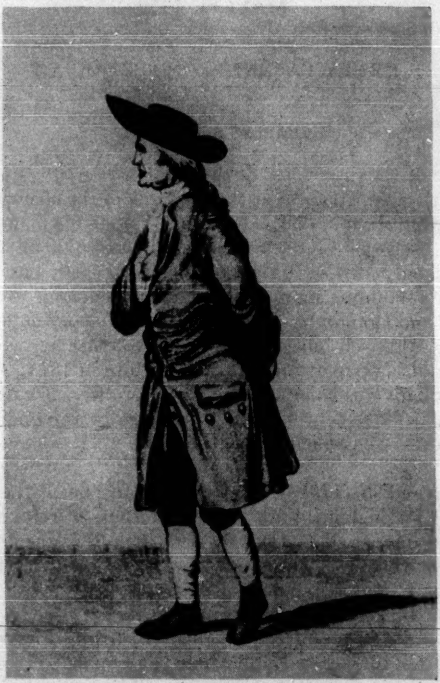
---

[1] Dunham Laboratory, Yale University, New Haven, Conn.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

Freed from the necessity of earning a living and, by virtue of his descent, from the necessity of establishing his name in the esteem of his fellowmen, he became even more of a misogynist than before. In fact, the meetings of the Royal Society and the scientific *conversaziones* at its President's house comprised nearly all his contacts with the outer world.

His disregard for popular opinion extended quite often into the field of scientific endeavor. Both Wilson, who edited his chemical papers, and Maxwell, who published his electrical researches, were struck with the casual way in which he made use of the law of constant proportion and the law of reciprocal combining proportion by weight in chemistry. His knowledge of these laws is most strongly implied in the notes on experiments performed early in 1777, when he made a "comparison of the conducting power of salt and fresh water"; and again in his 1788 paper "relative to the freezing of nitrous and vitriolic acids." In each



The only picture of Cavendish in existence, reproduced from a print which is in the possession of the British Museum.

case he stated the equivalent weights of the various chemicals he used quite as a matter of course, and the agreement with the figures nowadays quoted in any chemistry handbook is remarkable. Only the dates of these experiments bring the startling realization that Cavendish must have discovered these laws entirely for himself, years before the work of Wenzel, Richter, and Dalton was published, and simply neglected to tell anybody about it. He was satisfied in his own mind of their validity, and evidently saw no need for going beyond a mere statement of the laws. As a matter of fact, only one or two instances are known in which he found it necessary to perform an experiment before an audience of distinguished scientists, and then only in order to authenticate the truth of an experiment which others had been unable to repeat. On all other occasions, he was rather disdainful of his colleagues' judgment.

Of course, he might have had other scruples. His experiments on heat, made in 1764–65, were never published during his lifetime, except for a trifling fragment which appeared accidentally in his paper "On the congellation of quicksilver" (1783). It is evident from these experiments that Cavendish had discovered the fundamental laws of specific heat quite independently of Black, with the bulk of whose work he was not familiar. Had he published his results, he would doubtless been accorded precedence over Black; and Wilson, Cavendish's biographer, speculates: "Perhaps a reluctance to enter into even the appearance of rivalry with Black prevented him from publishing. . . . In truth, however, with Cavendish publication was the exception, not the rule, and . . . no special hypothesis is needed to account for [the papers] upon heat having remained in manuscript."

On the other hand, it is quite possible that Henry Cavendish was simply not used to having his word doubted. One of the two papers on electricity which were actually published during his lifetime deals with shocks obtained from an artificial model of the torpedo (an electric fish). In this paper, read before the Royal Society in 1775, Cavendish stated: "It appears from some experiments, of which I propose shortly to lay an account before this Society, that iron wire conducts about 400 million times better than rain or distilled water"; sea water, he found, conducted 100 times better than rain water, and a saturated salt solution 720 times better. As a matter of fact, he never did publish an account of these experiments—but his reputation was already so great that these figures were accepted immediately and presently others began using them in their computations. Such acceptance is even more curious if we realize that no method was known at the time by which electric resistances might be compared; over forty years were to elapse before the galvanometer would be invented.

How then did Cavendish make these measurements? It appears from his manuscripts that *he served as his own galvanometer*. He arranged several Leyden jars in a "battery" and, charging them equally, proceeded to discharge each jar individually through a circuit consisting of one of a group of calibrated tubes of liquid and his body in series. The intensity of the shocks he felt through his hands varied for various liquids; and by making many such measurements, he was able to compare the resistances of several media. He became so proficient at this technique that he was able to repeat his results with singular consistency; what is even more amazing, however, is that his results are in rather good agreement with modern measurements! One cannot but marvel at his ability as an experimenter—and yet wish that his capacity for interpretation had always been equal to it. For his curiosity led him almost at once to the next logical step: from the comparison of the resistances of various salt solutions he went on to investigate the manner in which the resistance of a given substance varied with the strength of the current passing through it. In modern terminology, the results of a series of four experiments were that the resistance varied as the

$$0.08,\ 0.03,\ -0.024,\ \text{and}\ 0.00$$

power of the current. These four figures—and especially the last one, which shows that resistance is independent of current—were, in effect, the first experimental proof of what we know today as Ohm's Law; yet Cavendish simply wrote it down in his journal and went on to other measurements. In fairness to him it should be noted that Ohm, having made his most famous discovery nearly fifty years later, also failed to appreciate its full importance; many more years had to elapse before scientists began to concern themselves with the *measurement* of resistance.

We have few clues as we try to divine, across the gap of a century and a half, why Cavendish so often delayed the publication of some of his most excellent experiments. His discovery of the composition of water is a case in point: he did not publish his results till three years after making the discovery, and in the meantime Lavoisier and Watt had both repeated Cavendish's experiment, published the results, and claimed priority. The claim made on behalf of Cavendish (for he would never have made one himself) is generally conceded to have carried the day; but the argument went on for decades, and a large part of Dr. Wilson's biography (published in 1851) is concerned with the "water controversy." It seems, however, that in this instance Cavendish withheld publication for the highest of scientific reasons—he was a trifle dubious about a certain aspect of his experiments. Some nitric acid had formed during the condensation process by which he sought to obtain pure water from hydrogen and oxygen, and further research was needed before he realized that his receptacle had been but imperfectly evacuated prior to the experiment; the small quantity of air which remained was the source of the nitric acid. But in the case of his work on electrical resistance, there was no such reason; yet Cavendish never published his results at all! Was it because he was ever more interested in opening up new fields and pursuing entirely new avenues of research? Or was he, in a day when each physicist had to be his own instrument designer and laboratory technician, more concerned with the purely theoretical aspects of scientific research? But no, he was most decidedly an experimenter, first and foremost. The published paper on electric fishes shows that; and quite a few of the other papers on electricity, though they remained in manuscript till long after his death, had been actually prepared for the printer—a fact which proves that he had meant to communicate his discoveries to the Royal Society ultimately. There is even evidence that he intended to publish them in book form. He just never got around to it. Moreover, the other paper on electricity which Cavendish did publish, "An attempt to explain some of the principal phænomena of electricity, by means of an elastic fluid," had firmly established his authority as an electrician; and in the following year he was appointed to a committee with the very practical task of providing protection against lightning for a government powder magazine. It is interesting to note that on this occasion Cavendish actually cooperated in an electrical investigation undertaken for the common weal with an even more celebrated electrician, who was the senior member of the committee: Benjamin Franklin.

It is thus evident that Cavendish was a practical physicist in the best sense of the term. Still, it has been suggested that he did have something of a mathematical complex. He seemed to derive the keenest intellectual pleasure from the processes of measuring and counting, and he was not above performing experiments with no other purpose than to count and measure. At the same time, he was a most accomplished mathematician. One need not read beyond the second page of the aforementioned paper on electrical theory to realize that he was perfectly conversant with higher mathematics and the calculus of Newton. It was in this paper that he stated his hypothesis that particles of the electric "fluid" repel each other with a force which is proportional to some power of the distance; he then went on to show mathematically, by the use of calculus, that this power will be the inverse square. His reasoning was strictly hypothetical: it was based on an assumption, to be proved or disproved at some future date, that the charge on a conductor resided entirely on the outer surface.

In 1785, the Frenchman Coulomb began publishing his wonderful series of papers on electricity and magnetism. He proved that the charge on a conductor resided on the surface, measured the repulsion and attraction of electrical charges by means of an ingenious torsion balance of his own design, and formulated the law of force, known to this day as Coulomb's Law, that the force between two electric charges is proportional to the product of the charges themselves, and inversely proportional to the square of the distance between them. It was generally supposed that Coulomb's work, which is the starting point of classical electromagnetic theory, had provided the experimental proof for Cavendish's hypothesis. Cavendish had read his paper to the Royal Society in 1771, and since he never followed up the theoretical discussion with experimental proof, the scientific world assumed that he deemed it unnecessary to pursue the matter further after Coulomb published his results. Imagine then Maxwell's surprise when, a century later, he found among Cavendish's manuscripts a complete account of an experiment proving that the charge of a conductor resided on its surface! This experiment had been performed in 1772, fully thirteen years before Coulomb made his entirely original contribution; and had Cavendish been a subject of the Czar, we would doubtless be now hearing clamors that Coulomb's Law should, after all, be properly known by the name of Cavendish.

Actually, the method employed by Cavendish in performing this important experiment differed so widely from Coulomb's procedure that the Frenchman's contribution would have been quite original even if he had known of the results obtained by Cavendish. Of more interest than the question of priority is the fact that Cavendish was the very first investigator to introduce the idea of potential, which he called "degree of electrification," as a quantity which was constant everywhere on a conductor. This concept was quite unknown to Coulomb and his contemporaries, and would have simplified their investigations and mathematical deductions considerably. Furthermore, Coulomb's Law in its modern form,

$$F = \frac{q_1 q_2}{\epsilon r^2}$$

takes into consideration the permittivity ϵ of the medium in which the charges q_1 and q_2 reside; this factor was omitted in the measurements made by Coulomb and Cavendish both. In fact, Cavendish merely established that

$$F \sim \frac{1}{r^2}$$

and Coulomb discovered that

$$F \sim \frac{q_1 q_2}{r^2}.$$

Nevertheless, Cavendish did observe that the charge between the tinfoil coatings on opposite sides of a plate of dielectric varied according to whether he used plate glass, crown glass, shellac, or bees' wax as the plate material. He even measured the specific inductive capacity (dielectric constant) of these substances, thus anticipating Faraday's discovery. In fact, says Maxwell, "the very idea of the capacity of a conductor as a subject of investigation is entirely due to Cavendish." This investigation also led to the experiment by which Cavendish proved the law of force.

He supported a metal globe, 12.1 in. in diameter, on a solid stick of glass; two hollow hemispheres, mounted on a hinged wooden framework in such a manner that they could be drawn together to form a slightly larger sphere, were situated so as to surround the first sphere completely, yet without touching it. Cavendish then made an electrical contact between the inner and outer sphere, charged the entire system, and broke the contact. Upon removing the outer sphere (by drawing apart the two hemispheres comprising it), he measured the charge on the inner sphere.

The description which Cavendish gives of this experiment is a beautiful example of scientific exposition. He was a very meticulous experimenter. The only device for the measurement of electric charge which was available to him was a pith ball electrometer, and he did not omit to make an estimate of the accuracy attainable with this simple instrument. After several trials, he concluded that a most careful measurement would reveal an increment or decrement in charge equal to one-sixtieth of that employed in his experiment. He found that he could perceive this minute variation in charge better if he imparted an initial charge to the electrometer, thus causing the pith balls to separate to a distance of about one inch before making any measurements. Yet when, after all these precautions, he applied the electrometer to the inner sphere, he could not perceive the slightest charge on it! Thus the entire charge was proved to reside on the outer sphere, and the one missing link in his hypothesis was supplied—doubtless to his great satisfaction. He had known it would turn out this way all along. All that remained to be done was to consolidate his results, write them down, and prepare the account of his experiment for the printer. But first he must perform some further experiments along the same lines. For instance, how well do the results apply to conductors having shapes other than spherical? And is the capacity of a conductor affected by the substance used, or by the place at which the charge enters the body? To these, and several other questions, Cavendish sought the answer before he would set down his results.

And set them down he did. The accounts of this series of experiments, which Cavendish actually made ready for publication, show quite clearly that he intended them to form a book. It is, therefore, doubly curious that Cavendish, who continued experimenting with electricity for another nine years, and lived on for another thirty-eight, never allowed these papers to go beyond the manuscript stage. But of one thing we can be sure: if published, his electrical researches alone would have assured him everlasting fame, even if he had not been the first to describe hydrogen, to discover nitric acid, to make a remarkably accurate calculation of the density of the earth, and to disprove the ancient misconception that water was an element.

Blood Transfusions Raise Man's High-Altitude Ceiling.—An extra pint of blood makes the difference in man's struggle to withstand critical altitudes, new experiments at Duke University have revealed.

Dr. Frank G. Hall, Duke physiologist and pioneer high-altitude experimenter, disclosed his findings at a recent UNESCO meeting of the world's top high-altitude specialists in Lima, Peru.

The value of blood transfusions in our tolerance of high-altitudes was proved this way at Duke:

Five student subjects were given a pint of blood, then tested in a special low-pressure chamber to see how "high" they could go. The result: All could tolerate about 2000 feet *more* altitude because of the extra oxygen they got from the blood.

Loss of blood worked the opposite way. Four students who gave blood were tested; they *lost* up to 2000 feet of altitude because of the loss of oxygen reserve they had given away in their blood.

Duke researchers measured tolerance by a special yardstick, "the interval of useful consciousness." As long as subjects were alert inside the chamber and mentally keen enough to do simple tasks and pass certain tests, they were considered "usefully conscious." In other words they performed tasks comparable to some of the things a pilot must do at 35,000 ft.

Midway in the tests they switched from breathing pure oxygen to inhaling the air circulating inside the chamber; meanwhile a constant pressure, equal to that fliers undergo at 35,000 ft., was maintained.

Tests were of the look-and-push variety. Symbols were flashed on a screen and subjects' reactions were tested by their ability to push matching keys. Meanwhile they breathed only normal air which was constantly decreasing in oxygen content.

Subjects who had the oxygen reserve in their bodies from the extra blood held out longer at the higher altitudes with the oxygen supply constantly decreasing.

Dr. Hall and his associates have been studying the effects of high altitude on man for many years at Duke. Dr. Hall was chief of the Army Air Forces' physiological branch during the war and wrote the first directive used in training U. S. air crews in the use of oxygen and oxygen equipment at high altitudes.

A new zenith camera for use in aeronautical chart-making has been developed for the Air Materiel Command and is now undergoing tests. Suitable at present for use in twilight or darkness, the camera is operated in true vertical, making images of stars for use in pin-pointing a ground location. Known position of the stars is compared with the spot from which the photograph was made, and a computation made therefrom.

A testing platform which simulates shipboard motion has been constructed at the Naval Aircraft Factory, Philadelphia, for studying the behavior of small self-propelled missiles which could be launched at sea. Designed to help study kinematic properties which might affect instruments in the missiles because of roll and pitch, this follows a Navy program of using land-mounted testing devices where accurate and practicable.

A NOTE ON THE SOLUTION OF CERTAIN APPROXIMATION PROBLEMS IN NETWORK SYNTHESIS *

BY

R. M. FANO[1]

ABSTRACT

This paper presents a method of determining appropriate functions to be used as amplitude characteristics of networks when certain ideal behaviors are to be approximated within specified tolerances. This method is based on the use of suitable transformations in the complex plane which, by simplifying the geometry of the problem, permit the direct determination by inspection of the desired functions. In the simplest cases of practical interest, this method has the only advantage of presenting results already known in a more intuitive form. On the other hand, it yields also alternative solutions satisfying additional and less conventional requirements and, at the same time, it indicates an approach likely to be fruitful in the solution of more complex approximation problems.

1. INTRODUCTION

The first step in the design of a network is the selection of an appropriate function to represent the particular frequency characteristic of interest. Such a function must, first of all, satisfy some necessary and sufficient conditions to insure the physical realizability of the network. In addition, it must meet whatever requirements are imposed by the design specifications. In many practical cases for instance, these design specifications require the function representing the behavior of the network at real frequencies[2] to approximate some simple step function within specified tolerances.

The approximation procedure described below has been developed (**1**)[3] in connection with functions representing the behavior of the magnitude of the transmission coefficient of a two-terminal-pair network terminated, on both sides, by pure resistances. However, it can be easily applied, with obvious modifications, to other types of amplitude responses, such as, for instance, the magnitude of a reflection coefficient or of a driving-point impedance. For the sake of simplicity, the discussion will be limited to the case of transmission coefficients, leaving to

---

* This work has been supported in part by the Signal Corps, the Air Materiel Command, and ONR. It was first published as Technical Report No. 62, Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Mass.

[1] Department of Electrical Engineering, Massachusetts Institute of Technology, Cambridge, Mass.

[2] When the complex variable $\lambda = \sigma + j\omega$ is used, "at real frequencies" means over the imaginary axis of the λ-plane.

[3] The boldface numbers in parentheses refer to the references appended to this paper.

the reader the extension of the approximation procedure to other similar cases.

Any function representing the transmission coefficient t of a passive two-terminal-pair network must satisfy the following necessary and sufficient conditions of physical realizability (**2**, **3**): (1) It must be a real rational function of the complex frequency variable $\lambda = \sigma + j\omega$; and (2) Its magnitude must satisfy the inequality $|t| \leq 1$ in the right half and on the $j\omega$-axis of the complex plane. It can be shown that the second condition is always satisfied if the poles of t are in the left half-plane and $|t| \leq 1$ on the imaginary axis.

The square of the magnitude of the transmission coefficient *on the imaginary axis* is seen to be equal to the value of the analytic function $T(\lambda) = t(\lambda)t(-\lambda)$ for $\lambda = j\omega$. The distribution of the zeros and poles of this new function $T(\lambda)$ must be symmetric with respect to the imaginary axis as well as to the real axis, and for physical realizability, the inequality $T(j\omega) \leq 1$ must be satisfied. It can be shown that, if $T(\lambda)$ meets these requirements, it is always possible to find at least one corresponding function $t(\lambda)$ which satisfies conditions (1) and (2) above. The determination of an appropriate function $T(\lambda)$ is just the problem with which this paper is concerned. The reader is referred to the literature (**1**, **2**, **3**) for the rest of the design procedure, namely, obtaining $t(\lambda)$ from $T(\lambda)$ and synthesizing the corresponding network. It must be pointed out, however, that considerations arising from these later stages of the design often play an important part in the selection of the function $T(\lambda)$. On the other hand, these considerations do not alter the main idea in the approximation procedure presented in this paper, and therefore will be disregarded in the following discussion.

2. THE ELECTROSTATIC POTENTIAL ANALOGY

The formal analogy between static field theory in two dimensions and complex function theory was recognized by the early workers in these fields. As a matter of fact, functions of a complex variable are commonly used in the solution of field problems. The reverse process has also been used successfully in the solution of network problems (**4**, **5**, **6**, **7**), because the field analogy provides convenient experimental techniques for the study of networks, and because the physical feeling for electrostatic fields can be very helpful as a guide in the selection of appropriate functions. This physical point of view will form the basis of the approximation procedure presented below.

The analogy between static fields in two dimensions and functions of a complex variable can be stated briefly as follows. The lines of constant real part and the lines of constant imaginary part of a function of a complex variable form in the complex plane two orthogonal families of curves, which can be identified with the equipotential lines and the lines of force of an electrostatic field. In particular, if one is interested in the magnitude of a function $F(\lambda)$, the real part of $\ln F(\lambda)$, that is $\ln|F(\lambda)|$, is identified with the potential; the constant phase lines become then the lines of force. If one turns his attention to the critical points (zeros and poles) of the type of function $F(\lambda)$ encountered in network theory (that is, the ratio of two finite polynomials), it is seen that these critical points become the sources of the corresponding electrostatic field. More precisely, one writes the function $\ln F(\lambda)$ in the form:

$$\ln F(\lambda) = \ln\left[A\frac{\prod_1^n(\lambda-\lambda_{0i})}{\prod_1^m(\lambda-\lambda_{pi})}\right] = \ln A + \sum_1^m \ln\frac{1}{\lambda-\lambda_{pi}} - \sum_1^n \ln\frac{1}{\lambda-\lambda_{0i}}, \quad (1)$$

where the λ_{0i} and the λ_{pi} are, respectively, the zeros and the poles of $F(\lambda)$, and A is a constant. It is clear then that the potential represented by the function $\ln|F(\lambda)|$ is produced, apart from $\ln A$, by positive and negative line charges of unit density located, respectively, at the poles and at the zeros of $F(\lambda)$. Consider now the approximation problem stated in the preceding section from this potential-theory point of view. This problem becomes that of determining a set of unit line charges which produces over an axis (the imaginary axis of the λ-plane) a potential distribution approximating a given function within a prescribed tolerance. An experimental technique for solving this problem has been developed by Hansen (4) using an electrolytic tank. A general theoretical solution, however, is not available at present and the procedure presented below should be considered as an attempt to fill this gap in a number of very simple but important cases. Because of the lack of generality of such a solution, the main idea can be best explained by means of specific examples. The following sections are devoted to such special cases.

3. UNIFORM PASS-BAND APPROXIMATION

Suppose it is desired to determine a function $T(\lambda)$ (representing the square of the magnitude of a transmission coefficient) which approximates a constant within a specified maximum deviation over the fraction of the imaginary axis corresponding to $-1 < \omega < 1$, and which approaches zero monotonically when ω approaches infinity. The reader will recognize these specifications as typical of low-pass filters from which corresponding high-pass, band-pass, and band-elimination filters can be obtained without difficulty.

Let, then, $T(\lambda)$ be the $F(\lambda)$ of the preceding section. Since $T(j\omega)$ is, by definition, real and positive, $\ln T(j\omega)$ is real. It will be remembered that $T(\lambda)$ is related to the transmission coefficient $t(\lambda)$ by $T(\lambda)$

$= t(\lambda)t(-\lambda)$. Following the line of thought discussed in the previous section, the problem at hand becomes that of determining a distribution of line charges which produces a potential function meeting the above specifications. The solution of this problem is made difficult by the fact that the potential is supposed to approximate a constant over only a portion of the imaginary axis. The problem would be quite simple, on the other hand, if the region of approximation extended over the whole axis.

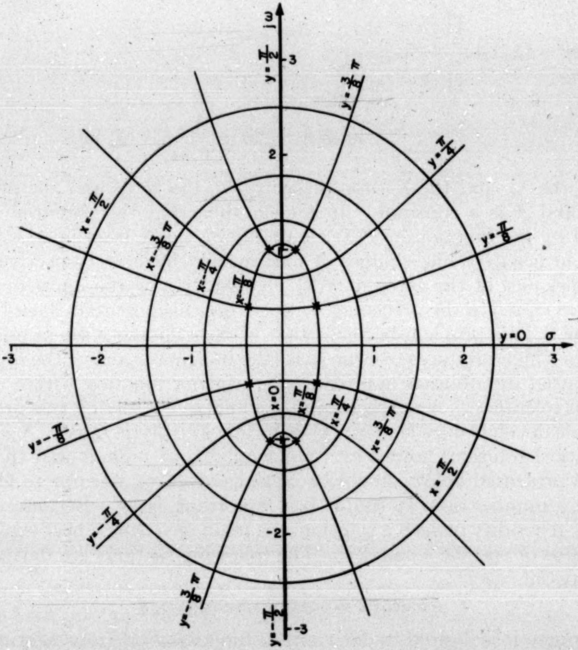


FIG. 1. The transformation $\lambda = \sinh z$ in the λ-plane. The points indicated with crosses correspond to the points indicated in a similar manner in the z-plane (Fig. 2).

The above remarks suggest immediately a method of approach which is currently used in field problems, namely that of simplifying the geometry of the problem by means of an appropriate transformation of the complex λ-plane. More specifically, in the case being considered, one wishes to find a function $\lambda = f(z)$, where $z = x + jy$, which transforms the segment $-1 < \omega < 1$ of the imaginary axis of the λ-plane into the whole imaginary axis of the z plane. It is readily recognized that the function $\lambda = \sinh z$ satisfies just this requirement. The straight lines x = constant become in the λ-plane a family of confocal ellipses, and the lines y = constant, a family of confocal hyperbolas. Figures 1 and 2 illustrate this transformation. It must be carefully noted, in this regard, that z is a multivalued function of λ, so that the whole λ-plane corresponds in the z-plane to an infinite number of regions, each one of which is limited by the straight lines $y = k\pi/2$ and $y = (k + 2)\pi/2$, where k is any positive or negative odd integer. The heavy lines in Fig. 1 are the branch lines of the function.

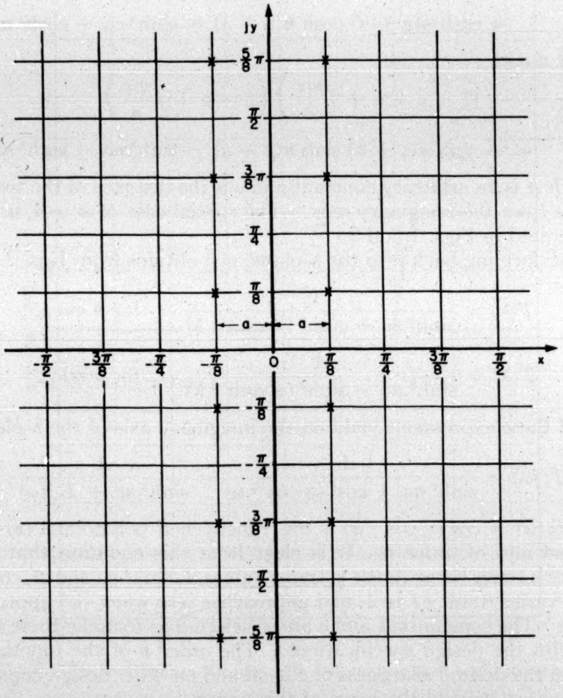

jy
$\frac{5}{8}\pi$
$\frac{\pi}{2}$
$\frac{3}{8}\pi$
$\frac{\pi}{4}$
$\frac{\pi}{8}$
a
a
$-\frac{\pi}{2}$
$-\frac{3\pi}{8}$
$-\frac{\pi}{4}$
$-\frac{\pi}{8}$
0
$\frac{\pi}{8}$
$\frac{\pi}{4}$
$\frac{3\pi}{8}$
$\frac{\pi}{2}$
x
$-\frac{\pi}{8}$
$-\frac{\pi}{4}$
$-\frac{3}{8}\pi$
$-\frac{\pi}{2}$
$-\frac{5}{8}\pi$

FIG. 2. The z-plane corresponding to the λ-plane of Fig. 1.

The solution of the approximation problem in the z-plane does not present any difficulty. Any uniform distribution of critical points over lines parallel to the imaginary axis will yield a function T with the desired type of oscillatory behavior over the y-axis. One must note, however, that, since $T(\lambda) \leq 1$ for $\lambda = j\omega$, no pole can be placed on any of the lines $y = k\pi/2$ (k is again any positive or negative odd integer), because these lines are transformed, in the λ-plane, into the part of the imaginary axis corresponding to $|\omega| > 1$. The simplest type of uniform distribution of critical points consists of two rows of simple poles parallel to the y-axis and symmetrically located with respect to it. If n is the number of poles in each row between any pair of lines $y = k\pi/2$ and $y = (k + 2)\pi/2$, one obtains, for n even:

$$\frac{A}{T(\lambda(z))} = \prod_{m=-\infty}^{\infty} \left(z + a + j\,\frac{\pi(1 + 2m)}{2n}\right)\left(z - a + j\,\frac{\pi(1 + 2m)}{2n}\right)$$

$$= \cosh n(z + a) \cosh n(z - a) = \sinh^2 na + \cosh^2 nz, \quad (2)$$

and for n odd:

$$\frac{A}{T(\lambda(z))} = -\prod_{m=-\infty}^{\infty} \left(z + a + j\,\frac{m\pi}{n}\right)\left(z - a + j\,\frac{m\pi}{n}\right)$$

$$= -\sinh n(z + a) \sinh n(z - a) = \sinh^2 na - \sinh^2 nz, \quad (3)$$

in which A is an arbitrary constant and a is the distance of the two rows of poles from the imaginary axis. The special case of $n = 4$, $a = \pi/8$ is illustrated in Figs. 1 and 2.

Transforming back into the λ-plane, one obtains from Eqs. 2 and 3,

$$T(\lambda) = \frac{A}{\sinh^2 na + \cosh^2 (n \sinh^{-1} \lambda)} \qquad \text{for } n \text{ even}, \quad (4)$$

$$T(\lambda) = \frac{A}{\sinh^2 na - \sinh^2 (n \sinh^{-1} \lambda)} \qquad \text{for } n \text{ odd}. \quad (5)$$

Both of these expressions yield on the imaginary axis of the λ-plane:

$$T(j\omega) = \frac{A}{\sinh^2 na + \cos^2 (n \cos^{-1} \omega)} = \frac{A}{\sinh^2 na + T_n^2(\omega)}, \quad (6)$$

where $T_n(\omega) = \cos (n \cos^{-1} \omega)$ is the Tchebycheff polynomial (8) of the first kind and of order n. It is clear from this equation that $T(j\omega)$ oscillates n times between the extreme values $A/\sinh^2 na$ and $A/\cosh^2 na$ when ω varies from -1 to 1, and approaches zero when $|\omega|$ approaches infinity. The constants A and a are so selected as to make these values agree with the design specifications. The order n of the function depends on the desired sharpness of cut-off and on other design considerations that are beyond the scope of this paper.

The location of the poles of $T(\lambda)$ in the λ-plane is obtained from the location of the poles in the z-plane. One has

$$\lambda_p = \sinh z_p = \begin{cases} \sinh\left(\pm a + j\,\dfrac{\pi(1 + 2m)}{2n}\right) & n \text{ even}, \\ \sinh\left(\pm a + j\,\dfrac{m\pi}{n}\right) & n \text{ odd}, \end{cases} \quad (7)$$

where m is any integer of a consecutive series of n integers. It is clear that the λ_p are located on an ellipse centered at the origin and of semi-axes equal to sinh a and cosh a.

The distribution of poles discussed above is the simplest distribution of critical points which yields an oscillatory behavior over the imaginary axis of the z-plane. The corresponding approximation function $T(\lambda)$ of Eq. 6 has been used by Darlington (2) and others (1, 3) as the magnitude squared of the transmission coefficient of low-pass filters. The function $-10 \log_{10} T(j\omega)$ is plotted in Fig. 3 for $n = 4$, $A = \sinh 4a$, and $a = 0.275$.

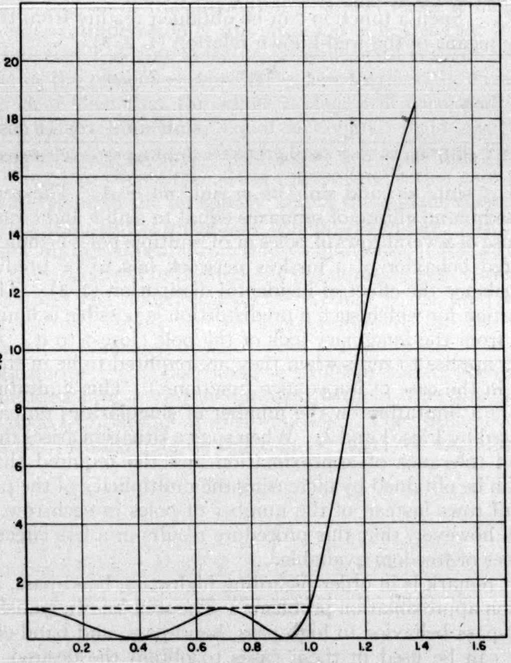


FIG. 3. The insertion loss of a filter with uniform pass-band approximation.

Approximation functions suitable for representing other network characteristics as well as reflection coefficients can be obtained by means of more complex distributions of critical points. One could use, for instance, more than one pair of rows of simple poles or multiple poles, and, in addition, one or more pairs of rows of zeros. Each pair of rows in the z-plane will transform, of course, into an ellipse in the λ-plane, so that all the critical points in the λ-plane will be located on confocal ellipses. The type of distribution of critical points that is to be selected in each particular case depends on the design specifications, on the form of network desired, and on the network characteristic that $F(\lambda)$ has to represent. For instance, if $F(\lambda)$ is to be the magnitude squared $|\rho|^2$ of the reflection coefficient of a low-pass filter, the number of zeros must be equal, at least, to the number of poles because $F(\lambda)$ must not vanish at infinity. Such a function can be obtained readily from the $T(j\omega)$ of Eq. 6 by means of the well-known relation (1, 2, 3)

$$|\rho|^2 = 1 - |t|^2 = 1 - T(j\omega). \tag{8}$$

One has then

$$|\rho|^2 = 1 - \frac{A}{\sinh^2 na + \cos^2 (n \cos^{-1} \omega)} = \frac{\sinh^2 nb + \cos^2 (n \cos^{-1} \omega)}{\sinh^2 na + \cos^2 (n \cos^{-1} \omega)}, \tag{9}$$

where $A \leq \sinh^2 na$, and $\sinh^2 nb = \sinh^2 na - A$. The zeros of $|\rho|^2$ are located on an ellipse of semiaxes equal to $\sinh b$ and $\cosh b$ (1).

The use of several rows of poles or of multiple poles is indicated when the desired behavior of a lossless network has to be predistorted to counterbalance the effect of incidental dissipation (2, 3). The amount of dissipation for which such a predistortion is possible is limited by the distance from the imaginary axis of the pole closest to it. (The same reasoning applies to zeros when they are required to lie in the left half-plane as in the case of impedance functions.) This limitation results, in turn, in a limitation on the number of singularities on each ellipse, as indicated by Figs. 1 and 2. When such a situation arises the required pass-band tolerance of approximation and the required sharpness of cut-off can be obtained by increasing the multiplicity of the poles or the number of rows instead of the number of poles in each row. It must be noted, however, that this procedure results in a less effective use of the degrees of freedom available.

A last remark is in order regarding high-pass, band-pass, and band-elimination approximation problems. The well-known transformations from low-pass behavior to high-pass, band-pass, and band-elimination behavior can be used in these cases to obtain the desired functions. However, these transformations are more conveniently performed directly on the network rather than on the function (3).

4. UNIFORM APPROXIMATION IN THE PASS-BAND AND IN THE REJECTION BAND

In many filter-design problems the requirements on the sharpness of cut-off and on the minimum rejection-band attenuation cannot be efficiently met by approximation functions of the type discussed in the previous section. In these cases, it is desirable to use, for the magnitude squared of the transmission coefficient $T(j\omega)$, a function which approximates in an oscillatory fashion a small constant in the rejection band, in addition to approximating a larger constant (smaller than unity) in the pass band. In most cases these two constants are made equal respectively to zero and one. One has now two distinct regions of approximation on the imaginary axis of the λ-plane: The pass-band region corresponding to $|\omega| \leq 1$ and the rejection-band region corresponding to $|\omega| \geq 1/k$, where the constant $k < 1$ is specified by the requirement on the sharpness of cut-off. The change of complex variable $\lambda = f(z)$ desired in this case must transform the pass-band and the rejection-band regions of the $j\omega$-axis into parallel straight lines in the z-plane. Closer consideration of this situation indicates that the transformation function $\lambda = f(z)$ must be periodic with respect to both x and y, while in the previous case sinh z was periodic only with respect to y. It is clear then that the whole λ-plane will correspond, in the z-plane, to an infinite number of equal rectangular regions such as the one limited by the two pairs of straight lines $x = \pm K$ and $y = \pm K'$, K and K' being two constants. It is convenient in this case to make the pass-band region of the imaginary axis of the λ-plane correspond to the real rather than to the imaginary axis of the z-plane. In this way the imaginary unit disappears from many equations. (The same situation would have occurred in Section 3 if the notation sinh z were not in common use and the notation $j \sin(-jz)$ had to be used instead.) It will also be convenient for the same reason to use the variable $w = u + jv = -j\lambda$ instead of λ.

The solution of the transformation problem stated above can be found in the literature or can be obtained without difficulty by means of the Schwarz-Christoffel transformation. The desired function $w = f(z)$ is the Jacobian elliptic function (9, 10, 11) of modulus k:

$$w = sn(z, k) = \sin \varphi, \tag{10}$$

where

$$z = \int_0^{\varphi} \frac{d\psi}{\sqrt{1 - k^2 \sin^2 \psi}} = \int_0^{w} \frac{dt}{\sqrt{1 - t^2}\sqrt{1 - k^2 t^2}}. \tag{11}$$

The constants K and K' are the complete elliptic integrals

$$K = \int_0^{\pi/2} \frac{d\psi}{\sqrt{1 - k^2 \sin^2 \psi}} \tag{12}$$

and

$$K' = \int_0^{\pi/2} \frac{d\psi}{\sqrt{1 - (k')^2 \sin^2 \psi}}, \tag{13}$$

where the complementary modulus k' is given by

$$k' = \sqrt{1 - k^2}. \tag{14}$$

This transformation is illustrated in Figs. 4 and 5. The heavy lines in Fig. 4 are the branch lines of the function.

By following a reasoning similar in principle to that used in Section 3, it can be seen that the critical points in the z-plane must be distributed uniformly along straight lines parallel to the axis or axes over which oscillatory approximation is desired. In this case, however, one has to use an infinite number of rows of poles and zeros because the transformation $w = sn\, z$ is doubly periodic. It will be noted that zeros as well as poles are required because the function $T(\lambda)$ must approximate zero or a small positive constant for $|\omega| \geq 1/k$. The simplest type of distribution of critical points satisfying these requirements consists of an infinite number of rows of poles and of zeros located in pairs at distances a and b, respectively, from the straight lines $y = 2mK'$. In most filter designs one makes $b = K'$ in which case the zeros of $T(\lambda)$ will be located on the imaginary axis of the λ-plane. The poles of $T(\lambda)$, on the other hand, must not lie on the $j\omega$-axis and therefore must not lie on the lines $x = \pm (2m + 1)K$ in the z-plane where m is any integer. Figure 5[4] illustrates the case in which the critical points in the same row are spaced a distance equal to $2K/4$. Figure 4 shows the location of the corresponding critical points in the w- and λ-planes, obtained from the relation

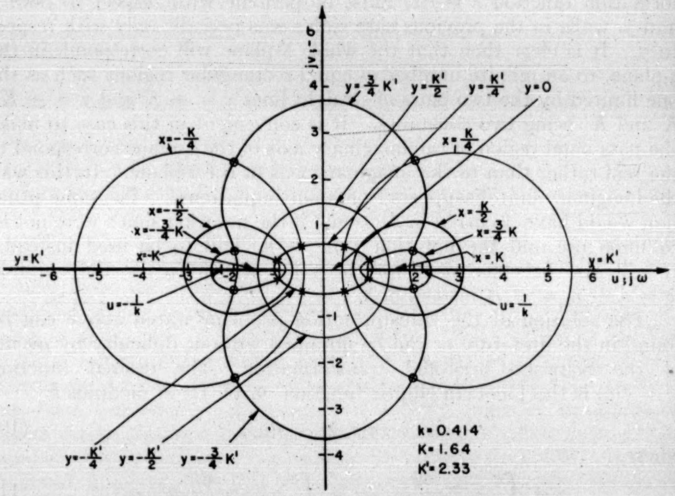


FIG. 4. The transformation $w = sn(z, k)$ in the w-plane. The points indicated with crosses and circles correspond to the points indicated in the same manner in the z-plane (Fig. 5).

$$w_i = -j\lambda_i = sn(z_i, k), \tag{15}$$

---

[4] The values of the parameters used in this figure ($k = 0.414$, $a = K'/4$, $b = 3K'/4$) are not representative of a practical design but lead to simpler and clearer graphical representations.

where z_i, w_i, and λ_i are the critical points in the z-, w-, and λ-planes, respectively.

To express in closed form the function whose critical points satisfy the above requirements, one observes first that the distribution of critical points for the function $sn^2(C_1z, k_1)$, where C_1 is a constant, consists of a rectangular array of double zeros and a rectangular array of double poles. Both arrays have spacings along the C_1x and C_1y axes equal, respectively, to $2K_1$ and $2K_1'$, one zero being located at the origin and one pole at the point $C_1z = jK_1'$. The constants K_1 and K_1' are the complete elliptic integrals of modulus equal, respectively, to k_1 and $k_1' = \sqrt{1 - k_1^2}$. This distribution of critical points is illustrated in Fig. 6. One observes further that the function considered has imaginary values over the lines $C_1x = mK_1$, where m is any integer. It follows

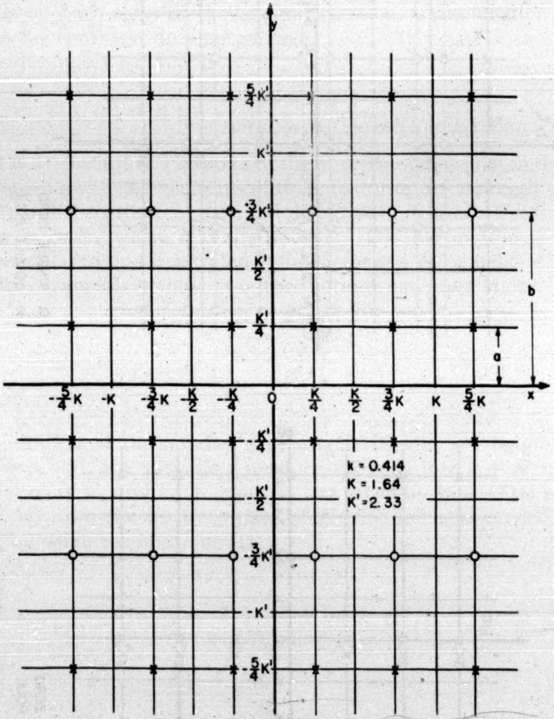

y
$\frac{5}{4}K'$
K'
$\frac{3}{4}K'$
$\frac{K'}{2}$
$\frac{K'}{4}$
b
a
$-\frac{5}{4}K$
$-K$
$-\frac{3}{4}K$
$-\frac{K}{2}$
$-\frac{K}{4}$
0
$\frac{K}{4}$
$\frac{K}{2}$
$\frac{3}{4}K$
K
$\frac{5}{4}K$
x
$-\frac{K'}{4}$
$k = 0.414$
$K = 1.64$
$K' = 2.33$
$-\frac{K'}{2}$
$-\frac{3}{4}K'$
$-K'$
$-\frac{5}{4}K'$

FIG. 5. The z-plane corresponding to the w-plane of Fig. 4.

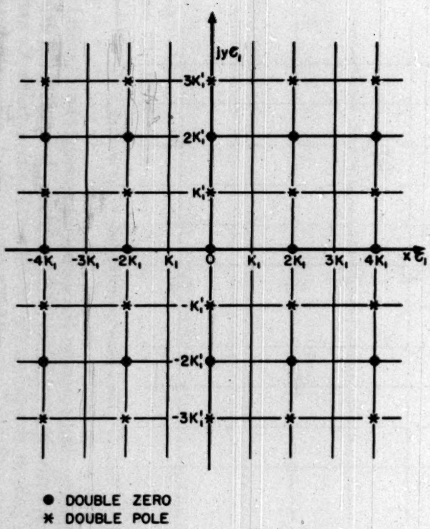

jyC₁
xC₁
3K₁'
2K₁'
K₁'
-K₁'
-2K₁'
-3K₁'
-4K₁
-3K₁
-2K₁
K₁
0
K₁
2K₁
3K₁
4K₁
● DOUBLE ZERO
* DOUBLE POLE

FIG. 6. Zeros and poles of the function $sn^2(C_1z, k_1)$.

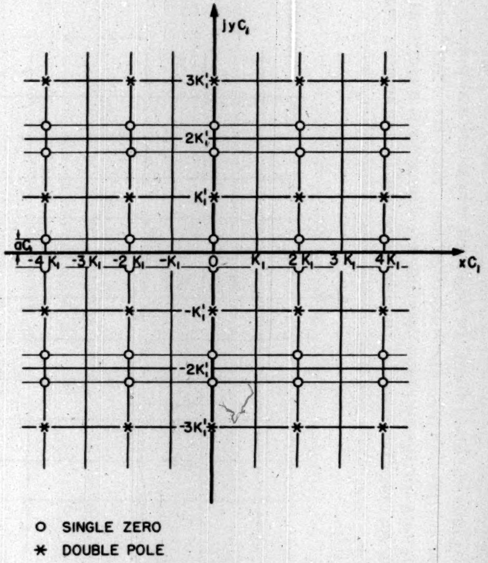

jyC₁
xC₁
aC₁
3K₁'
2K₁'
K₁'
-K₁'
-2K₁'
-3K₁'
-4K₁
-3K₁
-2K₁
-K₁
0
K₁
2K₁
3K₁
4K₁
O SINGLE ZERO
* DOUBLE POLE

FIG. 7. Zeros and poles of the function (23).

that the function

$$- sn^2(jC_1a, k_1) + sn^2(C_1z, k_1), \tag{16}$$

where a is a positive constant smaller than K_1', has the distribution of critical points illustrated in Fig. 7. It will be noted that the addition of a positive constant splits the rows of double zeros parallel to the real axis into rows of simple zeros displaced a distance C_1a along the imaginary axis on both sides of the original position. The location of the poles, on the contrary, does not change.

Consider finally the function

$$T[\lambda(z)] = A\left[\frac{- sn^2(jC_1b, k_1) + sn^2(C_1z, k_1)}{- sn^2(jC_1a, k_1) + sn^2(C_1z, k_1)}\right], \tag{17}$$

where A is a constant. The poles of the numerator cancel with the poles of the denominator and the zeros of the numerator are arranged in pairs of rows parallel to the real axis and displaced a distance C_1b from the lines $C_1y = 2mK_1'$ where m is any integer. The distribution of critical points is similar to that illustrated in Fig. 5, for $a = K'/4$ and $b = 3K'/4$. To identify completely these two distributions one must make

$$\left.\begin{aligned} \frac{K}{n} &= \frac{K_1}{C_1} \\ K' &= \frac{K_1'}{C_1} \end{aligned}\right\} \tag{18}$$

and in addition, if the number n of singularities in any length $2K$ of each row is odd, the variable z must be changed into $z + K/n$. The computation of C_1 and k_1 (K_1 and K_1' are given by tables (11) as functions of k_1) from the known values of K, K', and n is carried out as follows, by using the relations (12)

$$q_1 = e^{-\pi K_1'/K_1} = e^{-n\pi K'/K}, \tag{19}$$

$$k_1^{\frac{1}{2}} = \frac{\vartheta_2}{\vartheta_3} = 2q_1^{\frac{1}{4}}\frac{1 + q_1^2 + q_1^6 + q_1^{12} + \cdots}{1 + 2q_1 + 2q_1^4 + 2q_1^9 + \cdots}, \tag{20}$$

$$C_1 = \frac{K_1'}{K'} = n\frac{K_1}{K}, \tag{21}$$

where ϑ_2 and ϑ_3 are theta functions.

The next task is the determination of the tolerances of approximation in terms of the constants a, b, and A. For this purpose, one considers first the behavior of $T[\lambda(z)]$ on the real axis of the z-plane which corresponds to the segment $-1 < \omega < 1$ of the imaginary axis of the λ-plane. One observes that the function $sn(C_1x, k_1)$ is real and oscillates between the values 1 and -1, and that, if $b > a$, $- sn^2(jC_1b, k_1) > - sn^2(jC_1a, k_1)$. Therefore the function $T[\lambda(x)]$ oscillates between the maximum value

$$(T_{\max})_{|\omega|<1} = A \frac{sn^2\left(jn\,\dfrac{K_1}{K}\,b,\,k_1\right)}{sn^2\left(jn\,\dfrac{K_1}{K}\,a,\,k_1\right)} \tag{22}$$

and the minimum value

$$(T_{\min})_{|\omega|<1} = A \frac{1 - sn^2\left(jn\,\dfrac{K_1}{K}\,b,\,k_1\right)}{1 - sn^2\left(jn\,\dfrac{K_1}{K}\,a,\,k_1\right)} = A \frac{cn^2\left(jn\,\dfrac{K_1}{K}\,b,\,k_1\right)}{cn^2\left(jn\,\dfrac{K_1}{K}\,a,\,k_1\right)}, \tag{23}$$

where $cn(z, k_1) = \cos\varphi$, φ being given by Eq. 11. Standard trigonometric relations can be used to transform the above expressions into expressions more convenient for actual computations. It is to be noted in this regard that the tabulations available (11) give z as a function of φ for real values of φ.

The tolerance of approximation on that part of the imaginary axis of the λ-plane corresponding to $|\omega| > 1/k$ can be obtained by considering the behavior of T over the line $y = K'$. Then, using the relation (9)

$$sn(z + jK', k) = \frac{1}{k\,sn(z, k)} \tag{24}$$

one obtains immediately that the function T oscillates between a maximum value

$$(T_{\max})_{|\omega|>1/k} = A \frac{1 - k_1^2 sn^2\left(jn\,\dfrac{K_1}{K}\,b,\,k_1\right)}{1 - k_1^2 sn^2\left(jn\,\dfrac{K_1}{K}\,a,\,k_1\right)} \tag{25}$$

and a minimum value

$$(T_{\min})_{|\omega|>1/k} = A. \tag{26}$$

If one wishes to make $(T_{\min})_{|\omega|>1/k} = 0$ as is usually the case, the above formulas lead to a confusing limiting process which can be avoided by simply setting the numerator of Eq. 17 equal to one. This modification amounts to using the double poles of the denominator as zeros of the function. Equation 17 becomes then, for n odd,

$$T[\lambda(z)] = \frac{A}{-\,sn^2\left(jn\,\dfrac{K_1}{K}\,a,\,k_1\right) + sn^2\left(n\,\dfrac{K_1}{K}\,z,\,k_1\right)}. \tag{27}$$

For n even, z is changed into $z + K/n$. The limits of the two oscillations become

$$(T_{\max})_{|\omega|<1} = \frac{A}{-\,sn^2\left(jn\,\dfrac{K_1}{K}\,a,\,k_1\right)}, \tag{28}$$

$$(T_{\min})_{|\omega|<1} = \frac{A}{1 - sn^2\left(jn\,\dfrac{K_1}{K}\,a,\, k_1\right)} = \left[\frac{1}{A} + \frac{1}{(T_{\max})_{|\omega|<1}}\right]^{-1}, \tag{29}$$

$$(T_{\max})_{|\omega|>1/k} = \frac{k_1^2 A}{1 - k_1^2 sn^2\left(jn\,\dfrac{K_1}{K}\,a,\, k_1\right)} = \left[\frac{1}{k_1^2 A} + \frac{1}{(T_{\max})_{|\omega|<1}}\right]^{-1}, \tag{30}$$

$$(T_{\min})_{|\omega|>1/k} = 0. \tag{31}$$

The zeros and poles of $T(\lambda)$ are obtained directly from the corresponding critical points in the z-plane by means of Eq. 15. One has then for the poles

$$\left.\begin{aligned} \lambda_p &= j\, sn\left\{\left[(2m+1)\frac{K}{n} \pm ja\right], k\right\} && n \text{ even,} \\ \lambda_p &= j\, sn\left\{\left[2m\frac{K}{n} \pm ja\right], k\right\} && n \text{ odd,} \end{aligned}\right\} \tag{32}$$

and for the zeros

$$\left.\begin{aligned} \lambda_0 &= j\, sn\left\{\left[(2m+1)\frac{K}{n} \pm jb\right], k\right\} && n \text{ even,} \\ \lambda_0 &= j\, sn\left\{\left[2m\frac{K}{n} \pm jb\right], k\right\} && n \text{ odd,} \end{aligned}\right\} \tag{33}$$

where m is any integer of a consecutive series of n integers. When $(T_{\min})_{|\omega|>1/k} = 0$, $b = K'$ and therefore, using Eq. 24,

$$\left.\begin{aligned} \lambda_0 &= j\,\frac{1}{k\, sn\left[(2m+1)\dfrac{K}{n},\, k\right]} && n \text{ even,} \\ \lambda_0 &= j\,\frac{1}{k\, sn\left[2m\dfrac{K}{n},\, k\right]} && n \text{ odd.} \end{aligned}\right\} \tag{34}$$

The function $T(\lambda)$ can be written in terms of its critical points as follows:

$$T(\lambda) = B\,\frac{(\lambda^2 - \lambda_{01}^2)(\lambda^2 - \lambda_{02}^2)\cdots(\lambda^2 - \lambda_{0n}^2)}{(\lambda^2 - \lambda_{p1}^2)(\lambda^2 - \lambda_{p2}^2)\cdots(\lambda^2 - \lambda_{pn}^2)}, \tag{35}$$

where each factor represents two critical points with opposite signs. The value of B is determined by considering the behavior of the function when λ approaches infinity and therefore z approaches $\pm jK'$. One obtains without difficulty

$$\left.\begin{aligned} B &= (T_{\min})_{|\omega|>1/k} && n \text{ odd,} \\ B &= (T_{\max})_{|\omega|>1/k} && n \text{ even,} \end{aligned}\right\} \tag{36}$$

where the T's are given by Eqs. 25 and 26 or 30. However, for n odd and $b = K_1'$, Eq. 36 is meaningless because $T(\lambda)$ vanishes at infinity. One observes then that $T(\lambda)$ vanishes as B/λ^2 when λ approaches infinity (because one of the factors of the numerator of Eq. 35 is missing) and that $sn(z + jK', k)$ approaches infinity as $1/kz$ when z approaches zero. A little manipulation of Eq. 27 shows that

$$\lim_{\lambda\to\infty} T(\lambda) = \lim_{\lambda\to\infty} \frac{B}{\lambda^2} = \lim_{\lambda\to\infty} \left[-\left(\frac{k_1}{k} \frac{K_1'}{K'} \right)^2 \frac{A}{\lambda^2} \right]. \tag{37}$$

Therefore

$$B = -A \left(\frac{k_1}{k} \frac{K_1'}{K'} \right)^2. \tag{38}$$

It can be shown that Eq. 27 $\left(\text{for } n \text{ even substitute } z + \dfrac{K}{n} \text{ for } z\right)$ together with the transformation $\lambda = j\, sn(z, k)$ yields the approximation function used by many authors, for instance by Darlington (2). Figure 8 illustrates the behavior of the function $-10 \log_{10} T(j\omega)$ when $n = 4$, $(T_{\min})_{|\omega|>1/k} = 0$, $(T_{\max})_{|\omega|<1} = 1$, and $(T_{\min|\omega|<1} = 0.8$. Other more complex functions involving critical points of higher order or additional arrays of critical points can be constructed as in the case discussed in the previous section. It is felt that a detailed analysis of these functions will not add materially to the understanding of the design procedure.

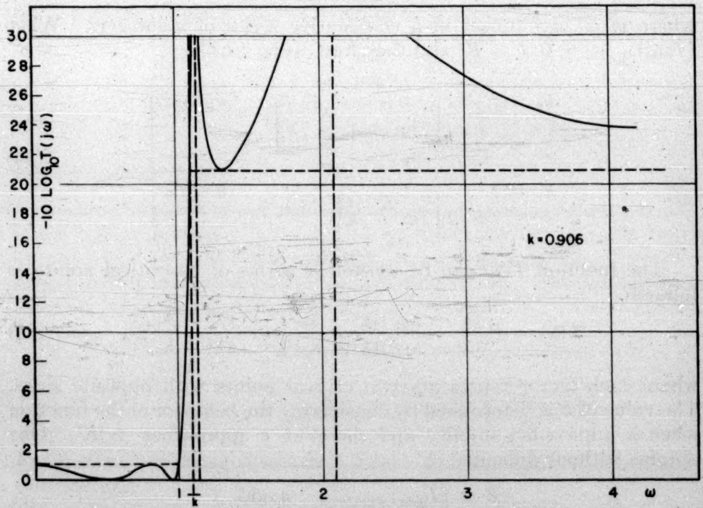


Fig. 8. The insertion loss of a filter with uniform approximation in the pass band and in the rejection band.

5. CONCLUSIONS

The examples presented in the preceding sections should be sufficient to make clear the ideas on which the approximation procedure is based. The field of application of such a procedure, on the other hand, is not limited to the examples discussed above although other cases have not been worked out in detail by the author. The usefulness of certain unconventional types of approximation functions in connection with the compensation of the effects of incidental dissipation was pointed out in Section 3.

A discussion of the synthesis procedure following the selection of the approximation function is beyond the scope of this paper. The reader is referred to Refs. 2 and 3 for such a discussion, and to Ref. 1 for a practical application of the approximation procedure presented above.

Acknowledgment

I wish to thank Dr. Manuel Cerrillo of the Research Laboratory of Electronics, Massachusetts Institute of Technology, for his generous mathematical help in connection with the manipulation of elliptic functions. Dr. Cerrillo was working at the same time as the author on the approximation problem discussed in Section 4, from a somewhat different point of view.

REFERENCES

(1) R. M. FANO, "Theoretical Limitations on the Broadband Matching of Arbitrary Impedances," JOURNAL FRANKLIN INST., Vol. 249, p. 57 and p. 139. Also summary: *Proc. Nat. Electronics Conference*, Chicago, November 3–5, 1947, pp. 109–118.

(2) S. DARLINGTON, "Synthesis of Reactance 4-Poles," *Jour. Mathematics and Physics*, Vol XVIII, 275–353, September 1939; also Bell Telephone System Monograph B-1186.

(3) R. M. FANO AND A. W. LAWSON, JR., "The Theory of Microwave Filters," Vol. 9, Ch. 9 (Edited by G. L. Ragan), Radiation Laboratory Series, McGraw-Hill, New York (1948).

(4) W. W. HANSEN AND O. C. LUNDSTROM, "Experimental Determination of Impedance Functions by the Use of an Electrolytic Tank," *Proc. I.R.E.*, Vol. 33, pp. 528–534, August, 1945.

(5) W. W. HANSEN, "On Maximum Gain-Bandwidth Product in Amplifiers," *J. App. Phys.*, Vol. 16, pp. 528–534, September, 1945.

(6) J. LINVILL, "An Experimental Approach to the Approximation Problem for Driving-Point and Transfer Functions," M.S. Thesis, M.I.T., 1945.

(7) W. H. HUGGINS, "A Note on Frequency Transformations for Use with the Electrolytic Tank," *Proc. I.R.E.*, Waves and Electrons Section, Vol. 36, pp. 421–424, March, 1948.

(8) B. VAN DER POL AND T. J. WEYERS, "Tchebycheff Polynomials and Their Relation to Circular Functions, Bessel Functions, and Lissajous Figures," *Physica*, Vol. 1, p. 78 (1933).

(9) A. C. DIXON, "Elliptic Functions," London and New York, The MacMillan Co., 1894, Appendix A.

(10) WHITTAKER AND WATSON, "Modern Analysis," New York, The MacMillan Co., 1946. Ch. XXII.

(11) P. R. E. JAHNKE AND F. EMDE, "Tables of Functions," New York, Dover Publications, 1943.

(12) WHITTAKER AND WATSON, *op cit.*, Sec. 21.61.

Synthetic Rubber Produced from Turpentine.—A new type of high-quality synthetic rubber, made with a chemical derived from turpentine, has been developed by scientists of the Bureau of Agricultural and Industrial Chemistry. This achievement, a result of work begun during the war by the Bureau's Naval Stores Research Division, makes it possible to use turpentine as a supplementary source of synthetic rubber in a national emergency. Under present conditions rubber from turpentine is somewhat more costly than GR-S, the most common synthetic rubber now produced commercially. But the new synthetic, compounded for use in tires, is slightly stronger than GR-S rubber, stretches better, and generates less heat under stress.

Main ingredient of the new elastomer is isoprene, a compound that forms the basic molecular unit of natural rubber. It is obtained from turpentine by a special molecule-splitting process developed by Bureau researchers. Isoprene produced from petroleum is already used in some types of synthetic rubber now on the market. The Bureau's method of producing it from turpentine should be a valuable national asset in the event of emergency shortages of petroleum.

First step in synthesizing rubber from turpentine is to produce isoprene from the turpentine's hydrocarbon constituents, called terpenes. Each molecule of isoprene has 5 atoms of carbon and 8 atoms of hydrogen. Terpene molecules are made of exactly twice this number of atoms—10 of carbon and 16 of hydrogen. Scientists of the Naval Stores Research Division devised a process for splitting the terpene molecule in half to obtain two molecules of isoprene.

This is done by immersing a conductor of high-resistance iron wire, heated red hot by an electric current, in boiling turpentine. The liquid vaporizes around this conductor and is heated to about 1400° F. When terpene molecules in the vapor strike the wire at this temperature they are split in two. The split molecules—isoprene—immediately bounce off into the liquid turpentine, which cools them sufficiently (even though it is boiling) to prevent further decomposition.

A continuous, two-stage distillation process removes the newly formed isoprene molecules from the turpentine bath and keeps them from again touching the red-hot wire. A water-cooled condenser first liquefies some of the vapors from the bath to unchanged turpentine, which is returned for further treatment. The remaining vapors, containing the isoprene, pass to a second condenser where they are liquefied with solid carbon dioxide ("dry ice"). This procedure gives a 70 per cent yield of isoprene that is about 95 per cent pure. It is made 99 per cent pure by redistillation.

In producing synthetic rubber, the purified isoprene (either alone or in combination with styrene, a compound used also in GR-S rubber) is mixed with water, soap, a catalyst, and a modifier, and shaken for a specified time at a warm temperature. One formula, found to yield approximately 80 per cent of high-quality synthetic rubber suitable for use in tires, calls for 75 parts (by weight) of isoprene, 25 parts styrene, 180 parts water, 5 parts soap, 3/10 part potassium persulfate (as a catalyst for the copolymerization of the isoprene and styrene), and 2/10 part mercaptan (which regulates the polymerizing action). This mixture is shaken for 14 hr. at a temperature of about 120° F. The resulting synthetic rubber has a tensile strength of about 3800 psi., will stretch to seven times its length, and in standard tests develops 7 to 18° F. less heat under stress than similarly produced and compounded GR-S rubber.

"DEVELOPABLE" AND DEVELOPED SILVER HALIDE GRAINS *

ATTEMPT AT A THEORY OF DEVELOPMENT OF GRAINS MADE "DEVELOPABLE" BY A GIVEN EXPOSURE. INTEGRATION OF THE SIMPLE FORMULAS OVER ALL GRAIN SIZES FOR THE CASE OF AN EXPONENTIAL SIZE-FREQUENCY FUNCTION

BY

LUDWIK SILBERSTEIN [1]

The number of grains of a photographic emulsion, of a given size a, made developable by any exposure $E = \frac{hc}{\lambda} n$ ** has been determined by probability considerations based on the quantum theory of exposure, that is, on the discrete structure of the incident radiation (light quanta, light darts).[2] For an ideally uniform emulsion (a equal for all grains), the number of "developable" grains per unit area, $k(E)$, thus determined, multiplied by the area a of each grain and by $M = \log e$, has been identified with what is commonly called photographic density, $D = Mk(E)a$.

Similarly, for an emulsion endowed with a size-frequency function $f(a)$ of any form whatever (for which the number of grains of size a to $a + da$ made developable by E may be denoted by $k(E)da$, we have identified the resultant photographic density with the product $Mak(E)da$ integrated over all sizes,

$$D = M\int_0^\infty ak(E)da. \tag{1}$$

This general expression was evaluated under the one-, two-, etc., generally, the r-quantic hypothesis, for exponential and "Gaussian" emulsions.

Manifestly, any of these expressions (subcases of (1)) can be claimed to represent the actual density or the logarithm of the opacity (due to exposure) only provided that *all grains* made "developable" by an exposure have actually been fully *developed*.[3]

---

* This paper was written as part of a larger work.

[1] Deceased; formerly Scientific Consultant, Rochester, N. Y.

** In this equation, h is Planck's constant, c is the velocity of light, λ is the wavelength and n is the number of incident quanta.

[2] L. Silberstein and A. P. H. Trivelli, *J. Opt. Soc. Am.*, Vol. 28, p. 441 (1938); Vol. 35, p. 93 (1945).

[3] And under the provisional (temporary) assumption that the sizes a' of the actually developed grains may approximately, and on the whole, be confounded with the sizes a of the original or only "developable" grains.

Whatever the nature of the developer, this condition can be satisfied only if the process of bathing in it the exposed emulsion is sufficiently protracted, that is, so that its further prolongation would not appreciably add to the density already obtained. In fine, the density D in formula (1) has to be replaced by

$$\lim_{t\to\infty} D(E, t),$$

if t be the development time. Practically, this limit may, to all purposes, be reached in something like a quarter of an hour, with, say, a DK-50 developer under ordinary conditions.

For materially shorter times t the characteristic curves, $D = D(E, t)$, with $\log E$ as abscissas, are well known to differ from the said limit, $D = D(E, \infty)$, not only as to vertical (that is, D-) dimension, but more or less as to shape also. That is to say, the quotient

$$D(E, t) : D(E, \infty)$$

is known (experimentally) not to be a mere function of t alone, but varying also from point to point, that is, a function of both the exposure and the development time.

The purpose of this paper is to make an attempt at a comparatively simple determination of this function of E and t, for any t.

Let, as above, $k(E)da$ be the number of grains ranging in size (area) from a to $a + da$ which have been made developable by an exposure E or $\frac{ch}{\lambda} n$ (where all symbols have the meaning previously explained). The photographic film or plate, thus exposed, being placed in a given developer, let us tentatively assume that the *probability*, p, that any one of these $k(E)da$ grains shall be "effectively hit" by a molecule or aggregate of molecules of the developer[4] is proportional to the development time t and to a, the size of the grain, say,

$$p = \kappa a t, \tag{2}$$

where κ will depend of course on the nature of the emulsion grain and of the developer, but may possibly be also a function of the exposure E. Then the probability that our grain shall be "effectively hit" at least s times (s or more times) is

$$1 - e^{-p} \sum_{m=0}^{s-1} \frac{p^m}{m!}.$$

Suppose now that the grain is actually developed whenever it is effectively hit at least s times; then the (probable) number of developed grains of size a to $a + da$ will be

---

[4] By "effectively hit" we mean only: subject to some (perhaps very complicated) act, not necessarily a sudden contact with a molecule, which contributes to an actual development, blackening of the grain.

$$k(E)\left\{1 - e^{-p}\sum_0^{s-1}\frac{p^m}{m!}\right\} da,$$

where p is as in (2).

Let $f(a)$ be the size-frequency function of the emulsion and N the total number of grains over unit area of the film, and therefore $Nf(a)da$ the number of grains of size a to $a + da$. Then, on the r-quantic hypothesis, the number of such grains made developable by an exposure $E = \frac{hc}{\lambda} n$ is

$$k(E)da = Nf(a)\left[1 - e^{-y}\sum_0^{r-1}\frac{y^m}{m!}\right],$$

where $y = na\epsilon$. Thus the number of developed grains of (original) size a to $a + da$ will be

$$Nf(a)\left[1 - e^{-y}\sum_0^{r-1}\frac{y^m}{m!}\right]\left[1 - e^{-p}\sum_{m!}^{p^m}\right] da,$$

and the resultant photographic density produced by the said exposure and a development during time t,

$$D(y, t) = MN\int_0^\infty af(a)\left[1 - e^{-y}\sum_0^{r-1}\frac{y^m}{m!}\right]\left[1 - e^{-p}\sum_0^{s-1}\frac{p^m}{m!}\right] da, \quad (3)$$

where $M = \log e \doteqdot 0.4343$, $y = n\epsilon a = \frac{\epsilon\lambda}{ch} E\cdot a$, and $p = \kappa ta$.

For $t \to \infty$ the last factor of the integrand tends to unity and the density to

$$D(y, \infty) = MN\int_0^\infty af(a)\left[1 - e^{-y}\sum_0^{r-1}\frac{y^m}{m!}\right] da, \quad (3a)$$

which is identical with the previous $D(y)$ obtained by the quantum theory of exposure.[2] If also the exposure E or y increases indefinitely, the last-written density tends to

$$D(\infty, \infty) = MN\int_0^\infty af(a)da = MN\bar{a}, \quad (3b)$$

where $\bar{a}$ is the over-all mean of grain sizes in the original emulsion. This limiting density, $MN\bar{a}$, will henceforth be denoted by D_m so that

$$D(\infty, \infty) \equiv D_m.$$

If, on the other hand, t has any finite value while $y \to \infty$, then the density tends to

$$D(\infty, t) = MN\int_0^\infty af(a)\left[1 - e^{-p}\sum_0^{s-1}\frac{p^m}{m!}\right] da, \quad (3c)$$

ultimately some function of the development time alone, which may be more conveniently denoted by $D_m(t)$. Thus,

$$D_m(t) \equiv D(\infty, t).$$

The formula (3) holds for any values of the positive integers r and s, and for any form of the size-frequency function $f(a)$ belonging to the emulsion in question.

Let us now consider, in all relevant detail, the special case of an "exponential" emulsion, for which, that is, $f(a) = e^{\frac{-a/\bar{a}}{\bar{a}}}$ or with $\bar{a}$ as our unit of all grain sizes,

$$f(a) = e^{-a},$$

and to begin with

$$r = 1 \quad \text{and} \quad s = 1,$$

that is, one-quantic hypothesis as to exposure and single-hit assumption as to development.

The general formula (3) reduces then to

$$D(y, t) = MN\int_0^\infty ae^{-a}(1 - e^{-n\epsilon a})(1 - e^{-\kappa t a})da$$

or, since $MN = MN\bar{a} = D_m$, to

$$\frac{D(y, t)}{D_m} = \int_0^\infty a\{e^{-a} - e^{-(1+n\epsilon)a} - e^{-(1+\kappa t)a} + e^{-(1+n\epsilon+\kappa t)a}\}da.$$

The first part of the integral is simply $\bar{a} = 1$, the next two terms amount to $-\frac{1}{(1 + n\epsilon)^2}$ and $-\frac{1}{(1 + \kappa t)^2}$, and the last is $\frac{1}{(1 + n\epsilon + \kappa t)^2}$.

Thus, and if we henceforth put

$$y = n\epsilon = n\epsilon\bar{a},$$

$$D(y, t) = D_m\left\{\frac{y(2 + y)}{(1 + y)^3} - \frac{y[y + 2(1 + \kappa t)]}{(1 + \kappa t)^2(1 + \kappa t + y)^2}\right\}. \tag{4}$$

The first term, identical with our previous $D_mE(y)$, depends on the exposure y alone, while the second term, which is always negative, depends on both variables, y and t. Equivalently, we may write

$$\frac{D(y, t)}{D_m} = E(y) - \frac{1}{(1 + \kappa t)^2} + \frac{1}{(1 + y + \kappa t)^2}. \tag{4'}$$

Here, the second term, a function of t alone, has the same value along the whole characteristic curve $(D, \log y)$, and the last term depends on both the development time and the exposure, namely, through the sum $\kappa t + y = \kappa t + n\epsilon a$.

For $t = 0$ (no development) we have $\frac{D}{D_m} = E(y) - 1 + \frac{1}{(1 + y)^2}$, that is, $D = 0$ for all y. For $t \to \infty$ the density tends to

$$D(y, \infty) = D_m E(y), \tag{4a}$$

the simple one-quantic function of exposure for an "exponential" emulsion. Here *all* grains made developable by the exposure y are developed, while for any finite t some of the "developable" grains remain yet undeveloped; accordingly,

$$D(y, t) < D_m E(y),$$

as shown directly by the original form, (4), of our equation.

For $y \to \infty$, and any finite development time, we have, by (4′),

$$D_m(t) \equiv D(\infty, t) = D_m \frac{\kappa t(2 + \kappa t)}{(1 + \kappa t)^2} < D_m, \tag{4b}$$

which may be written, compactly,

$$D_m(t) = D_m \cdot E(\kappa t).$$

The limiting density (at a fixed development time) is the same function of κt as $D(y, \infty)$ is of $y = n\epsilon$.

Keeping in mind the meaning of $E(y)$ and omitting the specification (y, t) of the symbol of density, we may write, for any exposure and any development time,

$$D/D_m = E(y) - \frac{1}{(1 + \kappa t)^2} + \frac{1}{(1 + y + \kappa t)^2}. \tag{4′}$$

The corresponding characteristic curve, D, $\log y$ with t = constant, will be called briefly a t-curve, for example, a 2-minute curve or 5-minute curve, etc., with, of course, a given developer.

If both D_m and the limiting density, $D_m(t)$, of an experimental t-curve are known, and if this curve can at all be well covered by the formula (4′), based on $f(a) = e^{-a}$, $r = s = 1$, then the required coefficient κ, a parameter of the function (4′), can be at once evaluated by means of the simple relation

$$E(\infty) - \frac{1}{(1 + \kappa t)^2} = 1 - \frac{1}{(1 + \kappa t)^2} = \frac{D_m(t)}{D_m},$$

which gives

$$\kappa = \frac{1}{t}\left[\sqrt{\frac{D_m}{D_m - D_m(t)}} - 1\right],$$

where, for any finite t, $D_m > D_m(t)$ and, therefore, $\dfrac{D_m}{D_m - D_m(t)} > 1$.

If, for instance, $D_m(t) = 3/4\, D_m$, then $\kappa t = \sqrt{4} - 1$ or $\kappa = 1/t$. Actually, our κt stands for $\kappa \bar{a} t$ so that the dimensions of the parameter κ are $[\kappa] = [(t^{-1})^{-2}]$.

The equation (4′) of the whole family of t-curves under investigation can be cast in an interesting form. Namely, the two terms to be subtracted from $E(y)$ amount to

$$\frac{1}{(1+\kappa t)^2}\left[1-\frac{1}{1+\left(\frac{y}{1+\kappa t}\right)^2}\right]$$

and the bracketed expression is just our function E with $\frac{y}{1+\kappa t}$ as argument. Thus our formula becomes

$$D/D_m = E(y) - \frac{1}{(1+\kappa t)^2}E\left(\frac{y}{1+\kappa t}\right). \tag{5}$$

The resultant t-curve is the difference of the $t = \infty$ curve, $D = D_m E(y)$ already familiar from the quantum theory of exposure,[2] and of the same curve shifted rigidly along the log y-axis through $\log(1+\kappa t)$ and with all its ordinates divided by $(1+\kappa t)^2$. Thus, for example, for $\kappa t = 1$ or 2 or 3 we have, respectively,

$$D/D_m = E(y) - 1/4\,E\left(\frac{y}{2}\right), \qquad E(y) - 1/9\,E\left(\frac{y}{3}\right),$$

$$E(y) - 1/16\,E\left(\frac{y}{4}\right),$$

and for $\kappa t = 1/2$,

$$D/D_m = E(y) - 4/9\,E(2/3\,y),$$

more generally, for $\kappa t = 1/m$,

$$D/D_m = E(y) - \frac{m^2}{(m+1)^2}E\left(\frac{my}{m+1}\right).$$

For $m \to \infty$, that is, for an evanescent development time, we have, after simple reductions,

$$D/D_m = \frac{2y(1+3y+y^2)}{(1+y)^3}\,\kappa t. \tag{5$_0$}$$

All ordinates of the curve are thus, in the limit, proportional to the development time. For $y \to \infty$ we have, in our previous notation, the limiting density

$$D_m(t) = 2\kappa t,$$

which follows also directly from (4b) for evanescent κt.

Returning to the general formula (5) and recalling that $E'(y) = 2/(1+y)^3$, we have, for the gradient $g = \frac{y}{M}D'(y)$ of the $D/\log y$

curve,

$$G(y) = \frac{2D_m}{M} y \left\{ \frac{1}{(1+y)^3} - \frac{1}{(1+y+\kappa t)^3} \right\}. \tag{6}$$

The necessary condition for its maximum ($g'(y) = 0$) becomes, with $z = 1 + y$, $\kappa t = \tau$,

$$\frac{1}{z^3} - \frac{1}{(z+\tau)^3} = 3(z-1)\left[\frac{1}{z^4} - \frac{1}{(\tau+z)^4}\right]$$

or

$$z(z+\tau)(3z^2 + 3z\tau + \tau^2) = 3(z-1)\{4z^3 + 6\tau z^2 + 4\tau^2 z + \tau^3\}, \tag{7}$$

a quartic for $z = 1 + y$. Its appropriate root will give $\bar{y} = z - 1$, the inflection point of the t-curve.

For $\tau \to \infty$ the equation reduces to $z = 3(z - 1)$, whence $z = 3/2$ or $\bar{y} = 1/2$, the well-known inflection point of the $t = \infty$ curve, $D = D_m E(y)$. For finite t and $\tau = \kappa t > 1$, we may develop the required root into a power series of the form

$$z = \frac{3}{2} + \frac{C_1}{\tau} + \frac{C_2}{\tau^2} + \frac{C_3}{\tau^3} + \cdots.$$

Substituting in (7) and comparing the coefficients of $\frac{1}{\tau}$, $\frac{1}{\tau^2}$, $\frac{1}{\tau^3}$, etc., we find $C_1 = 27/16$, $C_2 = -241/16$, and so on, so that the leading terms of $\bar{y}$ are

$$\bar{y} = \frac{1}{2} + \frac{27}{16\tau} - \frac{241}{16\tau^2} + \cdots; \tag{8}$$

if, say, $\tau = \kappa t$ is as large as 10, we may neglect the terms beyond that in $\frac{1}{\tau^2}$. This would give, to three decimals,

$$\bar{y} = 0.518,$$

but slightly greater than $\bar{y} = 1/2$, the inflection exposure of the $t = \infty$ curve. The reader may easily determine the next four coefficients and thus the terms in $1/\tau^3$, $1/\tau^4$, and so on, and thus be able to treat cases with $\kappa t < 10$, but always considerably greater than 1.

Practically, a much easier method of determining the position of the inflection point of any t-curve is, of course, the graphical method, *viz.*, drawing the curve of slopes (6) on a sufficiently large scale and picking out its maximum ordinate $g(\bar{y})$. It will suffice to give here one such example.

For $\kappa t = 1$ the factor of the constant $\frac{2D_m}{M}$ in (6) is

$$G(y) = y\left[\frac{1}{(1+y)^3} - \frac{1}{(2+y)^3}\right]. \tag{6.1}$$

This gives:

y	0.25	0.50	0.51	0.515	0.525	0.550	0.600	1	2	4
G	0.062	0.116	0.120	0.1158	0.1155	0.1150	0.112	0.088	0.428	0.0136

From this set of values, aided by a graph (not reproduced here) one finds that

$$0.500 < \bar{y} < 0.515.$$

To two decimals, we may safely accept

$$\bar{y} = 0.51$$

as the abscissa of the inflection point. So far, the case of $r = 1$ and $s = 1$.

Retaining the frequency function $f(a) = e^{-a}$ and the one-quantic hypothesis, let us still consider at some length the two-hit assumption,

$$s = 2.$$

We then have

$$D/D_m = \int_0^\infty a e^{-a}(1 - e^{-\nu})[1 - e^{-p}(1 + p)]da$$

or, again with y written for $n\epsilon$,

$$D/D_m = E(y) - \frac{1}{(1 + \kappa t)^2} E\left(\frac{y}{1 + \kappa t}\right) - 2\kappa t \left[\frac{1}{(1 + \kappa t)^3} - \frac{1}{(1 + y + \kappa t)^3}\right]. \tag{9}$$

The first two terms are as in (5), belonging to $s = 1$, so that

$$D_{s=2} = D_{s=1} - 2\kappa t \cdot D_m \left[\frac{1}{(1 + \kappa t)^3} - \frac{1}{(1 + y + \kappa t)^3}\right]. \tag{9'}$$

Thus, as could be expected offhand, $D_{s=2} < D_{s=1}$, for it is, manifestly, less probable to be "hit" twice than once.

As before, we have, for the $t = \infty$ curve, $D(y, \infty) = D_m E(y)$, while the limiting density for $y \to \infty$ and any finite t now becomes

$$D_m(t) = D_m \left\{E(\kappa t) - \frac{2\kappa t}{(1 + \kappa t)^3}\right\} = D_m \left\{1 - \frac{1 + 3\kappa t}{(1 + 3\kappa t)^3}\right\}. \tag{9a}$$

On the single-hit hypothesis we have had $D_m(t) = D_m E(\kappa t)$. The requirement of a second hit reduces the ratio $D_m(t) : D_m = E(\kappa t)$ by a subtractive term $2\kappa t/(1 + \kappa t)^3$.

The first task in attempting to represent a set of observed t-curves by our formula (9) is the numerical determination of the parameter κ. Now, if the ratio $\rho = D_m(t) : D_m$ is known from extrapolated density measurements, then by (9a)

$$(1 + \kappa t)^3 = \frac{1 + 3\kappa t}{1 - \rho}$$

is a *cubic* for κt, and thus for κ, the development time t being given. Put $1 + \kappa t = X$. Then the cubic for X will be

$$X^3 - \frac{3}{1 - \rho} X + \frac{2}{1 - \rho} = 0, \tag{10}$$

which is already of the normal form $X^3 + 3pX + 2q = 0$, with

$$p = - \frac{1}{1 - \rho}, \qquad q = \frac{1}{1 - \rho} = - p.$$

The familiar discriminant is

$$q^2 + p^3 = - \frac{\rho}{(1 - \rho)^3}$$

and since $\rho < 1$, this discriminant is always ***negative***, and the three roots of the cubic are all real, to wit,

$$X_1 = \frac{2}{\sqrt{1 - \rho}} \cos \frac{\theta}{3}, \quad X_2 = \frac{2}{\sqrt{1 - \rho}} \cos \frac{\theta + 2\pi}{3}, \quad X_3 = - (X_1 + X_2),$$

where θ is determined by $\cos \theta = - q(- p)^{-3/2} = - \sqrt{1 - \rho}$. Of these three values of $X = 1 + \kappa t$ the first only is positive and >1, thus leading to a positive κt. Ultimately, therefore, the procedure of determining the parameter κ from the observed ratio $\rho = D_m(t) : D_m$ is embodied in

$$\kappa t + 1 = \frac{2}{\sqrt{1 - \rho}} \cos \left[60^\circ - 1/3 \text{ arc cos } \sqrt{1 - \rho}\right]. \tag{11}$$

Let us apply this formula to the findings on an experimental emulsion, B-14, 4278A, recently investigated by A. P. H. Trivelli of Kodak Research Laboratories. With DK-50 at 65° F. as developer, the limiting densities for six different development times t (with the *minute* as our unit) are:

$t =$	2	3	4	5	8	12
$D_m(t) =$	0.43	0.51	0.62	0.70	0.88	1.10.

By near extrapolation we shall (tentatively) take

$$D_m \equiv D_m(\infty) = 1.20,$$

and, therefore,

$$\rho = 0.35 \qquad 0.425 \qquad 0.52 \qquad 0.58 \qquad 0.73 \qquad 0.922.$$

Substituting in (11) the first of these ρ-values we have

$$2\kappa + 1 = 2.48 \cos 48^\circ 31' = 1.66,$$

whence, to two decimals, $\kappa = 0.33$. The computation is by no means laborious. In much the same way the remaining five developments can be treated. The complete set of results is:

$t =$	2	3	4	5	8	12	
$\kappa =$	0.33	0.27	0.28	0.25	0.24	0.40;	$\bar{\kappa} = 0.295$
$\kappa - \bar{\kappa} =$	0.035	−0.025	−0.015	−0.045	−0.055	0.105.	

With the exception of the 12-minute development, the κ-values are fairly close to the over-all mean, $\bar{\kappa} = 0.295$.

If we attribute to κ_{12} a slightly smaller weight than to the remaining five κ-values (since $1 - \rho_{12} = 0.078$ only), and take, in round figures,

$$\kappa = 0.29, \qquad D_m = 1.20,$$

formula (9) becomes

$$\frac{D}{1.20} = E(y) - \frac{1}{(1 + 0.29t)^2} E\left(\frac{y}{1 + 0.29t}\right) - 0.58t\left\{\frac{1}{(1 + 0.29t)^3} - \frac{1}{(1 + y + 0.29t)^3}\right\}.$$

Thus, for example, for $t = 4$ (whose κ is closest to the adopted value, 0.29), the formula is

$$\frac{D(y)}{1.20} = E(y) - 0.214E(0.46y) - 2.32\left\{0.099 - \frac{1}{(y + 2.16)^3}\right\}.$$

It gives the following set of densities and their comparison with the observed ones, our theoretical point $D(1/4)$ being placed on the observed curve:

$y =$	$\frac{1}{32}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	∞
$D =$	0.05	0.10	0.18	0.305	0.45	0.58	0.64	0.66	0.67	0.67
Obs. =	0.04	0.105	0.20	0.305	0.42	0.52	0.595	0.62	—	—
$\Delta =$	0.01	0.005	−0.02	0.00	0.03	0.06	0.045	0.04		

The agreement is close enough (see also Fig. 1).

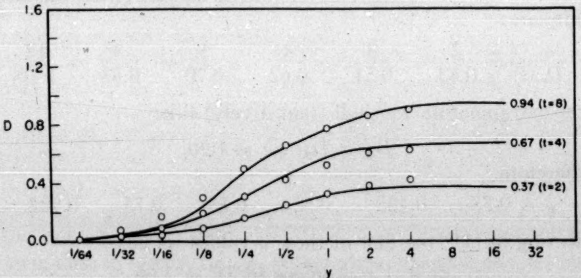


FIG. 1. B-14-4278A developed in DK-50 for 2, 4 and 8 minutes. Curves are calculated; ○○○, observed.

By placing the point $y = 1/4$ on the observed curve, we have correlated this value of y with $\log E = 0.63$ (the exposure E being quoted in ergs/cm.²), so that $y = 1$ corresponds to $\log E = 1.23$; briefly,

$$\log E_1 = 1.23.$$

Now, $y = \dfrac{\lambda\epsilon\bar{a}}{hc} E_1$, so that $\dfrac{\lambda\epsilon\bar{a}}{hc} = 1$, whence

$$\log \epsilon = \log \frac{hc}{\lambda\bar{a}} - \log E_1,$$

where $hc = 6.55 \times 10^{-27} \cdot 3 \cdot 10^{10} = 1.965 \cdot 10^{-16}$ in the c.g.s. system of units. Thus,

$$\log \epsilon = 0.293 - 16 - \log E_1 - \log (\lambda\bar{a}),$$

where λ and $\bar{a}$ are in cm. and cm.². If we henceforth write λ and $\bar{a}$ for the number of *microns* and *squared microns* (μ and μ^2) contained in the wavelength of the incident radiation and in the average grain size (area) of the emulsion, then (since $1\ \mu = 10^{-4}$ cm.) our formula becomes

$$\log \epsilon = \bar{4}.293 - \log E_1 - \log (\lambda\bar{a}). \qquad (12)$$

In the present case, $\lambda = 0.426$ and $\bar{a} = 0.56$, so that

$$\log \epsilon = \bar{4}.916 - \log E_1. \qquad (12a)$$

Thus, for the 4-minute development, when $\log E_1 = 1.23$, we find

$$\log \epsilon = \bar{5}.686 \quad \text{or} \quad (t = 4)$$
$$\epsilon = 4.85 \cdot 10^{-5}.$$

In much the same way the remaining five characteristic curves of the emulsion B-14-4278A have been treated. The results are shown graphically in Figs. 1 and 2 (which are self-explanatory) and in the following

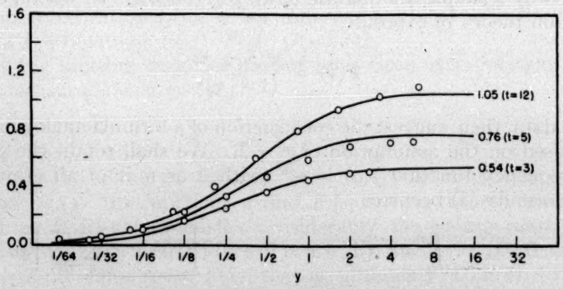


FIG. 2. B-14-4278A developed in DK-50 for 3, 5 and 12 minutes. Curves are calculated; ○○○, observed.

numerical table:

Emulsion B-14-4278A, developed in DK-50

$t =$	2	3	4	5	8	12
$D_m(t) =$	0.37	0.54	0.67	0.76	0.94	1.05
Obs. $=$	0.43	0.51	0.62	0.70	0.88	1.10
$D(1) =$	0.33	0.47	0.58	0.65	0.77	0.835
Log $E_1 =$	1.20	1.25	1.23	1.17	1.17	0.82
$\epsilon =$	5.2	4.6	4.85	5.6	5.6	(12.5) $\times 10^{-5}$

The totality of measurements embodied in the six experimental characteristic curves of the emulsion is fairly well represented by the $r = 1, s = 1$ formula (9), based on the schematic size-frequency function $f(a) = e^{-\frac{a/\bar{a}}{a}}$, with the parameters

$$\kappa = 0.29, \qquad D_m = 1.20.$$

As will be seen from Figs. 1 and 2, serious deviations, $\Delta = D - D_{\text{obs.}}$, occur only in the lower portions of the 8-minute and the 12-minute curves. The logarithm of the exposure correlated with $y = 1$ oscillates irregularly between $\log E_1 = 1.17$ and 1.25 for the first five developments and so does the intrinsic sensitivity ϵ, which is inversely proportional to E_1. The mean of the first five ϵ-values is

$$\bar{\epsilon} = 5.17 \cdot 10^{-5}. \qquad \text{(B-14-4278A)}$$

This exhausts, essentially, the discussion of the observations made on the said emulsion, as an illustration of the formula (9) based on the *one*-quantic hypothesis[2] and the "single-hit" assumption ($r = 1, s = 1$).

Another emulsion, N4X-13318, of average grain size $\bar{a} = 0.81\ \mu^2$, exposed, as before, to $\lambda = 0.426\ \mu$ and developed in ferrous oxalate during 2, 3, 4, 6, 9, and 18 minutes at 65° F., has its 18-minute curve well covered by a simple *two*-quantic term, $D(y) = D_m F(y)$, where, as in the quantum theory of exposure,

$$F(y) = \frac{y^2(3 + y)}{(1 + y)^3}.$$

These data, then, suggest the construction of a formula analogous to (9), but based on the assumption of $r = 2$. We shall retain the previous size-frequency function $f(a) = e^{-a}$ (with $\bar{a}$ as unit of all grain sizes). Thus formula (3) becomes

$$D/D_m = \int_0^\infty a e^{-a}[1 - e^{-y}(1 + y)]\left[1 - e^{-p} \sum_0^{s-1} \frac{p^m}{m!}\right] da, \qquad (13)$$

where $p = \kappa t a$. It remains to fix the value of the integer s. For $y = \infty$, and any t, we have, with the abbreviation $D(\infty, t) = D_m(t)$,

$$\frac{D_m(t)}{D_m} = \int_0^\infty ae^{-a}\left[1 - e^{-\kappa ta}\sum_0^{s-1}\frac{(\kappa ta)^m}{m!}\right]da$$

and since

$$\int_0^\infty ae^{-a}da = 1,$$

$$\frac{D_m(t)}{D_m} = 1 - \int_0^\infty ae^{-(1+\kappa t)a}\sum_0^{s-1}\frac{(\kappa ta)^m}{m!}\,da.$$

Thus, for $s = 1$,

$$\frac{D_m(t)}{D_m} = 1 - \frac{1}{(1+\kappa t)^2} = E(\kappa t),$$

as before. For $s = 2$,

$$\frac{D_m(t)}{D_m} = \frac{\kappa^2 t^2(3+\kappa t)}{(1+\kappa t)^3}$$

or, in the notation of the quantum theory,[2]

$$\frac{D_m(t)}{D_m} = F(\kappa t).$$

For $s = 3$,

$$\frac{D_m(t)}{D_m} = F(\kappa t) - \frac{3!\kappa^2 t^2}{2(1+\kappa t)^4} = \frac{\kappa^3 t^3(4+\kappa t)}{(1+\kappa t)^4}$$

or

$$\frac{D_m(t)}{D_m} = G(\kappa t).$$

In fine, for $s = 1, 2, 3$, etc., the ratio $p = D_m(t):D_m$ is equal to the simple *one*-quantic, *two*-quantic, *three*-quantic, etc., function with κt (instead of y) as argument.

Thus, in order to settle the value of s, we have only to compare the set of observed ratios ρ (for a few different t-values) with the sets of values of the functions $E(\kappa t)$, $F(\kappa t)$, $G(\kappa t)$, etc.

Now the limiting densities formed by a close extrapolation of the highest measured densities are:

$t =$	2	3	4	6	9	18
$D_m(t) =$	0.99	1.19	1.22	1.25	1.28	1.39.

An inspection of the functions E and F (quantum theory) shows that this set of $D_m(t)$-values misfits considerably the values *const.* $E(\kappa t)$, while it agrees fairly with the set of *const.* $F(\kappa t)$, namely, if we correlate with $t = 2$ the value *const.* $F(2)$, that is, ultimately, $D_mF(2)$. In fact, if we put $D_mF(2) = 1$, that is to say, $0.741\ D_m = 1$ or $D_m = 1.35$, then we have the set of numbers

$$D_mF(2) = 1, \quad D_mF(4) = 1.21, \quad D_mF(8) = 1.30, \quad D_mF(16) = 1.34,$$

which approximates the observed set of $D_m(t)$ closely enough.

This correlation (of $t = 2$ with $\kappa t = 2$) implies

$$\kappa = 1 \qquad \text{and} \qquad D_m = 1.35$$

and at the same time, $s = 2$.

Thus, formula (13) becomes

$$D/D_m = \int_0^\infty ae^{-a}\{1 - e^{-y}(1 + y)\}\{1 - e^{-\kappa ta}(1 + \kappa ta)\}da, \quad y = n\epsilon a,$$

or, after simple integrations, and with $n\epsilon = n\epsilon\bar{a}$ ultimately denoted by y (as before),

$$D/D_m = 1 - \frac{1}{(1 + y)^2} - \frac{2y}{(1 + y)^3} - \frac{1}{(1 + \kappa t)^2} - \frac{2\kappa t}{(1 + \kappa t)^3} + \frac{1}{(1 + y + t)^2} + \frac{2(y + \kappa t)}{(1 + y + \kappa t)^3} + \frac{3y\kappa t}{(1 + y + \kappa t)^4}$$

or

$$D/D_m = F(\kappa t) + F(y) - 1 + \frac{1 + 4(y + \kappa t) + 3[(y + \kappa t)^2 + y\kappa t]}{(1 + y + \kappa t)^4}. \qquad (14)$$

This formula implies $r = 2$, $s = 2$ and, as before, $f(a) = e^{-a}$.

In our present case, $D_m = 1.35$ and $\kappa = 1.00$.

Let us now compare the corresponding particular formula,

$$D/1.35 = F(y) + F(t) - 1 + \frac{1 + 4(y + t) + 3[(y + t)^2 + yt]}{(1 + y + t)^4} \qquad (14a)$$

with the six characteristic curves observed on the emulsion in question.

In the first place we have for $y = \infty$, $D_m(t) = 1.35F(t)$, i.e., for

$t =$	2	3	4	6	9	18
$D_m(t) =$	1.00	1.14	1.21	1.28	1.31	1.34
Observed =	0.99	1.19	1.22	1.25	1.28	1.39
$\Delta =$	0.01	−0.05	−0.01	0.03	0.03	0.05

The agreement is close enough.

For $t = 18$, $F(t) - 1 = -0.009$ and, for $y = 1$, the last term in (14*a*) amounts only to 0.007, so that

$$\frac{D(1)}{D_m} = F(1) - 0.002 = 1/2 - 0.002.$$

Similarly we have, to three decimals,

$$\frac{D(2)}{D_m} = F(2) - 0.002, \qquad \frac{D(1/2)}{D_m} = F(1/2) - 0.003.$$

In fine, for $t = 18$, to three decimals, we have simply

$$D(y) = 1.35F(y).$$

This gives

$y =$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16	32
$D =$	0.01	0.06	0.14	0.35	*0.675*	1.00	1.21	1.29	1.34	1.35
$\log E_1 = 0.60,$										
$D_{\text{obs}} =$	0.00	0.02	0.10	0.32	*0.675*	1.00	1.22	1.32	1.36	1.38
$\Delta =$	0.01	0.04	0.04	0.03	0.000	0.00	−0.01	−0.03	−0.02	−0.03

Thus, the $t = 18$ curve is represented closely enough by formula (14*a*). See also Fig. 4, where the continuous curve is calculated and the observed densities are marked by eyelets. The same figure contains also the results derived from (14*a*) with $t = 3$ and 6. The analogous results for the remaining three curves $t = 2$, 4, and 9 are represented graphically in Fig. 3. The agreement between the calculated and the observed curves is, with the exception of the shoulder of the 2-minute curve, remarkably close.

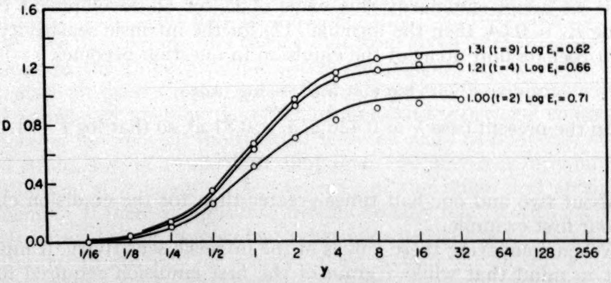


FIG. 3. N4X-13318 developed in FE OX for 2, 4 and 9 minutes. Curves are calculated; ○○○, observed.

The correlation of the y-variable with the exposure E (in ergs/cm.^{2}) adopted in our charts (Figs. 3 and 4) for the six curves is:

$t =$	2	3	4	6	9	18
$\log E_1 =$	0.71	0.65	0.66	0.60	0.62	0.60
$E_1 =$	5.1	4.5	4.6	4.0	4.2	4.0

The six values of E_1, that is, of E corresponding to $y = 1$, though chosen independently of each other, do not differ much from their

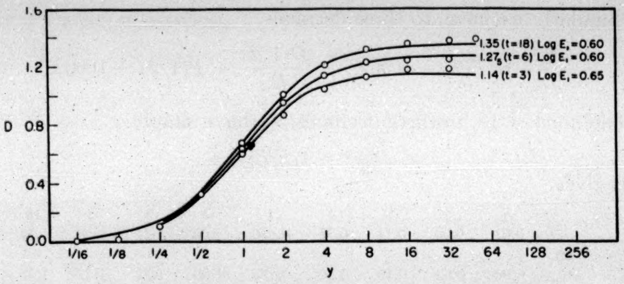


FIG. 4. N4X-13318 developed in FE OX for 3, 6 and 18 minutes. Curves are calculated; ○○○, observed.

arithmetical mean,

$$E_1 = 4.4 \text{ (erg/cm.}^2\text{)}.$$

The deviations from the mean are:

$$+0.7 \qquad +0.1 \qquad +0.2 \qquad -0.4 \qquad -0.2 \qquad -0.4.$$

If we adopt summarily this value of E_1 for all development times, or $\log E_1 = 0.64$, then the formula (12) for the intrinsic sensitivity of a grain (per its unit area) of the emulsion in question becomes

$$\log \epsilon = \bar{5}.65 - \log (\lambda \bar{a}).$$

In the present case $\lambda = 0.426\ \mu$, $\bar{a} = 0.81\ \mu^2$, so that $\log \epsilon = \bar{4}.11_2$ or

$$\epsilon = 1.30 \cdot 10^{-4}$$

or about two and one-half times greater than for the emulsion chosen for our first example.

When comparing these values of the intrinsic sensitivity, it must be kept in mind that while a grain of the first emulsion required for its developability but *one* quantum ($r = 1$), a grain of the second emulsion called for $r = 2$ quanta, so that its effective sensitivity comparable with that of the first emulsion is $\epsilon/2 = 6.50 \times 10^{-5}$, not much greater than 5.2×10^{-5}.

These two examples ought to suffice for the illustration of the general exposure-development formulas derived in this paper.

We have treated, each time, a single quantic exposure term combined with a development term. In much the same way, the exposure formulas based on a superposition (or mixture) of two quantic terms can be combined with expressions corresponding to development. The structure of the development terms or factors may, in general, be different for the two (or more) components of a superposition.

THE PART OF OCTAHEDRAL THEORY IN THE STUDY OF THE PLASTICITY OF METALS *

BY

WENDELL P. ROOP [1]

WHAT IS THE OCTAHEDRAL THEORY?

To this query it is not quite enough to answer: go and read twenty-five pages in the *Welding Journal* (**1**)[2]; that would make a rather heavy piece of home work. At the same time short and simple statements about intricate matters can always be shown to be wrong. We will therefore begin with some short phrases which have been used in this connection.

Discussion has recently occurred about "all deformation theories," namely, "those which postulate that the components of elastic and permanent strain are completely determined by the existing components of stress" (**2**).

The expression "deformation theory" has also been stated to embody the idea that "stress and strain are uniquely related as long as loading continues" (**3**). This may be a less exacting condition than the earlier one. We may require of a theory that from given intensities and orientations of the three principal stress components we shall be able to calculate all the details of the strain. Or we may be content with partial knowledge, such as the effect of moderate transverse components, otherwise ignored, on the relation between axial components of stress and strain, say, in a tube test. Or, again, we may seek only numerical evaluation of a function (say energy) of the stress and strain components. If these components are functionally related with each other, such an evaluation is possible; otherwise, perhaps not. But it is less ambitious to ask only for energy values than for a complete solution of the problem.

In any case, deformation theories certainly include the octahedral theory, for one of its oversimplified and sometimes tacit assumptions is that "a stress-strain curve exists." This is mathematical jargon for the supposition that if a curve is plotted to show how stress and strain are related with each other in a simple tension test, calculation will suffice for deriving a corresponding relation for other states of stress combinations. If this were not true, the more detailed unique relation between existing stress and strain referring to intensity and orientation of each of the separate components would surely be illusory.

---

* An address to the technical staff of the Lewis Laboratory, National Advisory Committee for Aeronautics, Cleveland, Ohio, February 10, 1949.

[1] Structural Research Laboratory, Swarthmore College, Swarthmore, Pa.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

Another way of putting it is to ask whether the material has an "equation of state," as persons with a physical-chemical background like to say. For our present purpose this phrase also will be taken to refer to the correctness of using simple tension test data for inferences about what to expect when other stress components besides simple axial tension are also present.

Such an assumption does not seem very risky to most engineers, since they have often been content with much less than a complete stress-strain curve—such information, for example, as that given by a load-elongation curve; or perhaps even with single values of yield-point, ultimate strength, and elongation in a standard specimen.

At the very next sitting of the Applied Mechanics Division of the ASME after the discussion of deformation theories (see ref. 2), a detailed calculation was presented (4) and accepted with no adverse criticism, in which the assumption indicated above was so obviously considered to be valid that it was not even mentioned.

At the same time, cases can be imagined in which it is equally obvious that the changes in stress and in strain have a character quite different from those in a simple tension specimen. Such a case is pure torsion of a tube or bar of circular section in which the directions of the principal stresses remain unchanged with reference to the cylinder, whereas axial lines ruled on the metal are subject to a continuous rotation as the strain advances. While the state of stress at a point in the metal thus loaded remains in a sense unchanged (if there is no strain hardening) yet the strain changes continually; there can thus be no relation between them like that in the tension test.

Now, if the octahedral theory is an approximation in the sense that it gives solutions of plastic problems that are nearly if not exactly true, then before we can find out how great the error may be we must somehow have the exact truth with which to compare the approximation. In the torsion test, however, the situation is not like that, since a deformation theory not only gives a result which needs correction, but it has no meaning at all. It is not wrong, it is just not applicable. If we are going to make use of octahedral theory we have to keep clear of cases in which deviation of the principal axes of strain from those of stress is more than a very moderate amount. It may be hard to identify such cases; but if the octahedral theory gives conclusions reasonably agreeing with observation we are reasonably free from such difficulties.

THE BASIC EQUATIONS

These are as follows:

$$\tau_1/\gamma_1 = \tau_2/\gamma_2 = \tau_3/\gamma_3, \tag{A}$$

$$\epsilon_1 + \epsilon_2 + \epsilon_3 = 0, \tag{B}$$

$$\tau = \tau(\gamma), \tag{C}$$

where

τ designates shear stress generally, and in particular octahedral shear stress,

γ designates shear strain generally, and in particular octahedral shear strain, and

$\epsilon_1\epsilon_2\epsilon_3$ are the principal strains, taken parallel to the greatest, the middle, and the least principal stresses.

In addition to the stress-strain curve, Eq. *C*, we have thus set up two other conditions, constant volume in the material, Eq. *B*, and equality of shear stress-strain ratios, Eq. *A*. All three of these relations are subject to error which affects the precision of conclusions from the theory, but errors of this kind can be accepted on the ground that a result subject to correction is better than no result at all.

Now these equations are quite different from the equations of equilibrium and compatibility; they do not at all serve to replace those monuments of mathematical majesty. They presuppose the same concepts of the nature of the stress tensor and the strain tensor as are used in elastic theory, and these are sufficiently described for the present purpose by the graphic constructions of Mohr (1).

What was originally wanted was a criterion for the passage of the material outside the elastic range so that it might be known within what limits the elastic theory was still valid. By a curious confusion of thought it seemed that when we could no longer depend on our calculations the safety of our structures must be in danger. The expressions which were devised to indicate the limit of validity of a theory were regarded also as criteria of material failure.

What saved the structures was a strongly empirical attitude. Though bridges and ships were designed to work within elastic limits, stress concentrations were ignored, as was of course also the ductile action which removed their curse. Now that we are very conscious of these concentrations and are even beginning to understand something of their extent as well as of their intensity, we must also know more of what happens beyond the yield point.

What we now want to do is to calculate behavior in the plastic range. It happens that certain relations which were devised as criteria of yield (5) are also highly significant for flow (6). Plastic behavior may be described in terms of certain functions of the principal strains and of the principal stresses. The simplest examples of such functions are the octahedral shear strain (γ) and the octahedral shear stress (τ). These have some useful properties which will presently be discussed; but first let us see how they are related to Condition (*A*).

The octahedral value (of strain or stress) is essentially the root mean square of the principal shear components.[3] Its nature (in case of strain) is conveniently exhibited by giving to the ratio of the intermediate to the highest of the three principal strains the single symbol

---

[3] Any function significant for plastic flow in the desired way must be invariant with respect to orientation of the reference frame of co-ordinates at a point in the metal. In the equation

$$m = \epsilon_2/\epsilon_1.$$

Then by Condition (*B*)

$$-(1+m) = \epsilon_3/\epsilon_1$$

and for the octahedral strain

$$\tfrac{3}{2}\gamma = \sqrt{\gamma_1^2+\gamma_2^2+\gamma_3^2} = \sqrt{6}\sqrt{1+m+m^2}\,\epsilon_1.$$

Since for stress no condition like (*B*) is applicable, two ratio symbols are needed:

$$n_2 = \sigma_2/\sigma_1 \qquad \text{and} \qquad n_3 = \sigma_3/\sigma_1.$$

Then for the octahedral stress τ,

$$\tfrac{3}{2}\tau = \sqrt{\tau_1^2+\tau_2^2+\tau_3^2} = \frac{1}{\sqrt{2}}\sqrt{1-(n_2+n_3)+n_2^2+n_2n_3+n_3^2\,\sigma_1}.$$

Now if we give the symbol γ to the ratio

$$\tau_1/\gamma_1 = \tau_2/\gamma_2 = \tau_3/\gamma_3 = r$$

we also have the octahedral values in the same ratio

$$\tau/\gamma = r.$$

This ratio changes with the level to which plastic flow has advanced in accord with Condition (*C*); it is the secant modulus of the stress-strain curve. The secant modulus at a given level is the same for any of the corresponding shear components as for the octahedral values.

THE BASIC IDEAS

The essence of the octahedral theory is this: that the basic stress-strain curve is that which relates with each other the *octahedral* values. We are in the habit of giving attention only to the greatest or the axial components of strain and of stress. When the transverse components are ignored, their variation has the baffling effect of upsetting the axial stress-strain curve. Thus, for given octahedral values, the axial values are given by the above expressions.

In general, octahedral theory does not suffice to define a relation between the stress ratios n_2 and n_3 and the strain ratio m; the theory refers only to the shear values, and changes can be made in load which will affect the principal stress ratios and yet not affect the shear stress.

---

relating principal components with components taken parallel to the co-ordinates of an arbitrary reference frame, certain coefficients appear which are called "*the* invariants." It happens that octahedral values are closely related to the coefficient which is called the second or quadratic invariant.

Such a change would be the addition of a uniform hydrostatic pressure or tension. Only when some additional condition is set can this theory give determinate solutions about relations between stress and strain components at a fixed point in the metal.

GRAPHIC METHODS

The relations defined by the theory are graphically exhibited by the Mohr diagrams which have been shown in (1). The essential theorem asserts that the Mohr diagrams for stress and for strain are geometrically similar to each other. Similarity in form of these diagrams leaves the position of the stress circles undetermined with respect to the origin.

Another graphical procedure which has had extensive use under special conditions is that of the phase diagram; in a typical form this makes use of a phase-space in which the co-ordinates are values of the three principal stresses at a single point in the geometrical space. The history of a given loading sequence is depicted by a line in the phase space which passes from each to the next of a series of loading conditions. As these conditions succeed each other in time each point on the curve is associated with an instant of time. Or the scheme may be generalized by using another variable instead of time which designates the level of loading; for this variable the symbol t may also be used. It is monotonic with respect to time.

A function f of the principal stresses is now imagined, like the octahedral stress, or any other function which may happen to be chosen for a special physical significance. Contoured surfaces of constant f may then be established in the phase space.

Now start from a point at which the principal stresses are $\sigma_1 = a$, $\sigma_2 = b$, $\sigma_3 = c$; the principal strains at this point I will call $\epsilon_1 = A$, $\epsilon_2 = B$, $\epsilon_3 = C$. Now increase each of the principal stresses in the same ratio, $(1+\delta$ to $1)$, where δ is a small fraction, say 1 per cent. Then in a great number of cases of plastic flow all the principal strains will also increase by a different fraction, say Δ. At different levels the ratio Δ/δ will vary and at very high strain values, such as when the fracture point is approached, the axes may not remain parallel; but in many practical cases the conditions I have described will continue to hold up to fairly large strains, say 10 or 20 per cent, or even higher. The path in the phase space will be one on which the ratios m and n remain constant, and these are radial lines drawn through the starting point. Perhaps this is an arbitrary starting point in that it ignores residual stresses and pre-strain history. If there is a difference between a given sample and one in which residual stress and pre-strain have been removed by heat treatment, then this will need separate investigation.

But it is certain that there are circumstances under which the assumed conditions are satisfied and "a strain function exists." Several sets of data have been taken, notably in tube tests under various constant ratios of internal pressure to axial tension, all of which lead to the same curve (within experimental error) of octahedral stress on octahedral strain. A limited number of trials have been made (7) to impose a variation on the ratios m and n in the course of a tube test and so obtain a moderate departure from the radial path in the phase diagram; within the limits of scatter in the results, no effect on the end-point could be observed.

Later we will have a look at some specific examples of the use of the phase space, but first let us survey quickly the kinds of problems in which help might reasonably be expected from such a resource as octahedral theory.

TYPES OF PROBLEMS

The classical problem of elastic theory is that of inferring the internal patterns of stress distributions from known patterns of external load applied to a body. To accomplish this same result when the functions are no longer linear and the strains no longer infinitesimal is so far still impossible. This has led some persons to assert that there is now no such thing as a theory of plasticity, and others to suspect that in this sense there never will be.

But the matter need not stop there. If, beyond the yield point, strain can advance at a point in the metal with no increase in stress at all (perfect plasticity) some useful inferences can be drawn from assumptions about strain patterns, as has been done in the theory of "plastic bending" (**4, 8, 9, 10**).

Quite different is the problem of designing external constraints (dies) for guiding plastic flow so that loads high in the plastic range may produce desired end-forms in the metal without fracture (forming limits). Some headway has been made on this line with theory (**11, 12**) though it is doubtful how much use is made of it. However even the most empirical procedures must be informed by some sort of idea as to why metals behave as they do in a die or in a press. Octahedral theory is an idea so simple that generally it is not given a thought and since it takes no account of anything but homogeneous and isotropic material, practical men may not be conscious of its value. Yet it does have the merit of stating in form suitable for calculation a set of ideas which can give good guidance where its lack of precision is not a drawback (**13**).

The capacity for absorbing damaging energy by plastic flow is for some types of structure always, and for most types of structure occasionally, of critical importance. By study of the residual strain distribution in a model or service structure after damaging overload, the parts which made the most effective contributions to its survival or salvage value may be identified. For this purpose octahedral theory gives us exactly what we want in a formula relating energy absorption with plastic strain directly, obtained by integrating Eq. C.

THIN-WALLED STRUCTURES

From this point onward attention will be given only to the condition of "plane stress," in which one of the principal stresses is zero. This condition is closely satisfied in structures with thin walls, and nowadays that means most structures of metal.

Under these conditions the usefulness of octahedral theory is greater than in the more general case, since it now gives us explicit relations between the components of stress and those of strain. Thus, from the point of view of the Mohr circles, the stress diagram is now no longer free, but is anchored to the origin.

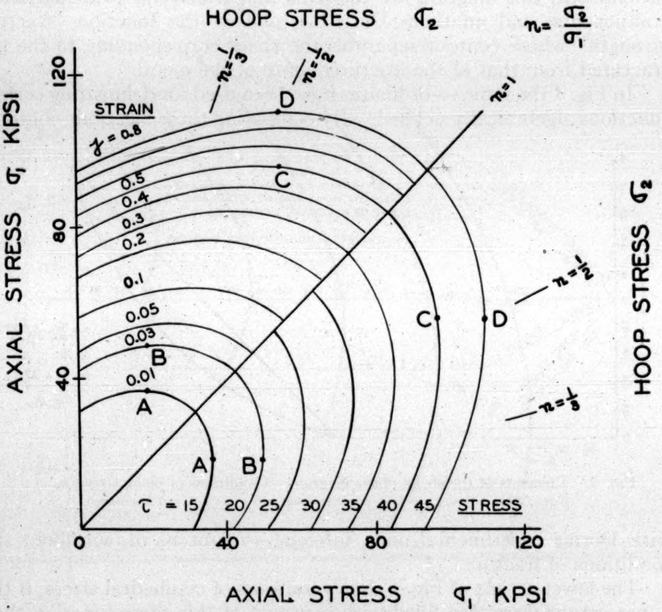


FIG. 1. Contours of constant octahedral stress τ and of constant octahedral strain $\gamma = \left(\frac{\tau}{47.6}\right)^4$ as observed for ship steel.

In the algebraic treatment of the subject it is now no longer necessary to use two different values of n, the ratio of transverse to axial stress. Of n_2 and n_3 one is now zero and we may ignore it and apply the single symbol n to the other one. We will not bother to write down again the simplified forms to which the various equations are now reduced, but pass directly to graphic charts illustrating the result. In doing this we will use the phase diagram, and now that the number of phases has been reduced to two, it is convenient to use the third co ordinate in our familiar space for the function *f*, whatever it may be. It will be delineated by contours along which the function has constant specified values.

GRAPHIC OCTAHEDRAL DATA

The first such diagram is a very familiar one; it has been used recently for exhibiting experimental data on fracture (**14**). The co-ordinates in this diagram are the axial and transverse principal stress components, and on them Dorn has plotted the function "fracture strength" whose contour separates the space corresponding to the unfractured from that of the fractured state of the metal.

In Fig. 1 the same co-ordinates have been used for delineating certain functions algebraically defined. By comparing these algebraic contours with Dorn's experimental ones, inferences might be drawn about the conditions of fracture.

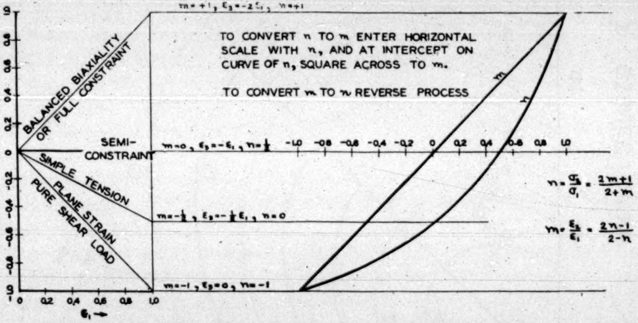


FIG. 2. Elements of the strain phase diagram. Conditions of plane stress, $\sigma_3 = 0$.
NOTE: When $n < 0$, a special convention about subscripts 1, 2, and 3 is needed.

The lower octant of Fig. 1 shows contours of octahedral stress; if the upper octant were also filled with contours of this same function they would be mirror images of those actually shown, with symmetry about the line on which *n* is unity. Instead of that, the upper octant is used to show contours of octahedral strain calculated by the parabolic formula

$$\tau = 47.6\gamma^{\frac{1}{4}} = 53.8(1 + m + m^2)^{-\frac{1}{8}}\epsilon_1^{\frac{1}{4}}$$

which is generally applicable to ship steel. The successive contours correspond to advancing levels of plastic action.

From this familiar stress-phase diagram we now turn to another phase-diagram which for some purposes is even more useful, in which the co-ordinates are strains rather than stresses. Such a diagram illustrates especially well the conditions under which inferences are to be drawn from directly observed strain values such as are obtainable from the grids, which have been rather widely used recently (7).

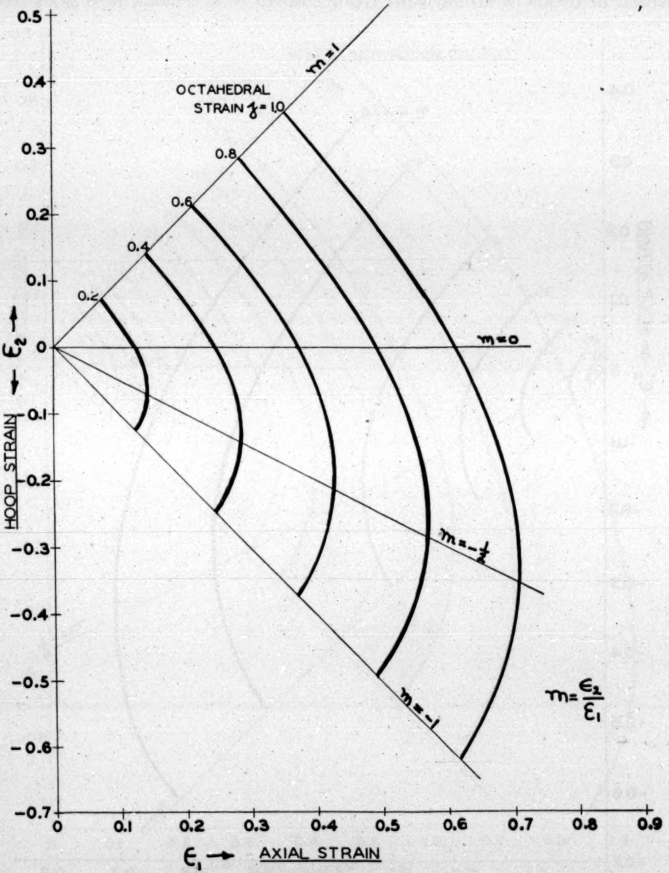


FIG. 3. Contours of octahedral strain (under biaxial stress, $\sigma_3 = 0$). As in a tube test plotted on the axial and hoop strains, ϵ_1 and ϵ_2.

Figure 2 shows the relation between the stress ratio n and the strain ratio m. A phase diagram is set up on the left, and the conditions which correspond to various diagonal paths are noted; these differ from the conditions at the same path-angles in the stress phase diagram. Shifting from stress to strain has the effect of shifting sectors between radial paths by variable amounts in the clockwise direction. The octant which in terms of stress went from 1:30 to 3:00 o'clock now goes from

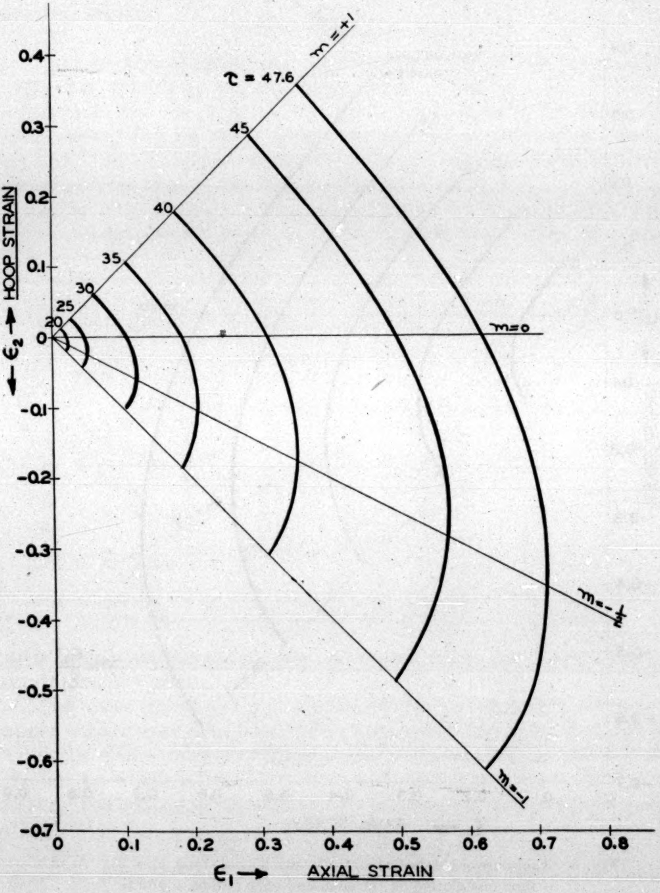


FIG. 4. Contours of octahedral stress ($\sigma_3 = 0$), plotted on axial and hoop strains.

1:30 to 3:45. On the strain diagram negative values of the transverse component must also be considered, so that the diagram is extended to a full quadrant from 1:30 to 4:30.

At the abscissa where axial strain is unity a vertical line is drawn and the intercepts of the diagonals on this line identify the values of m used in the right side of the diagram. Here the value of n is plotted

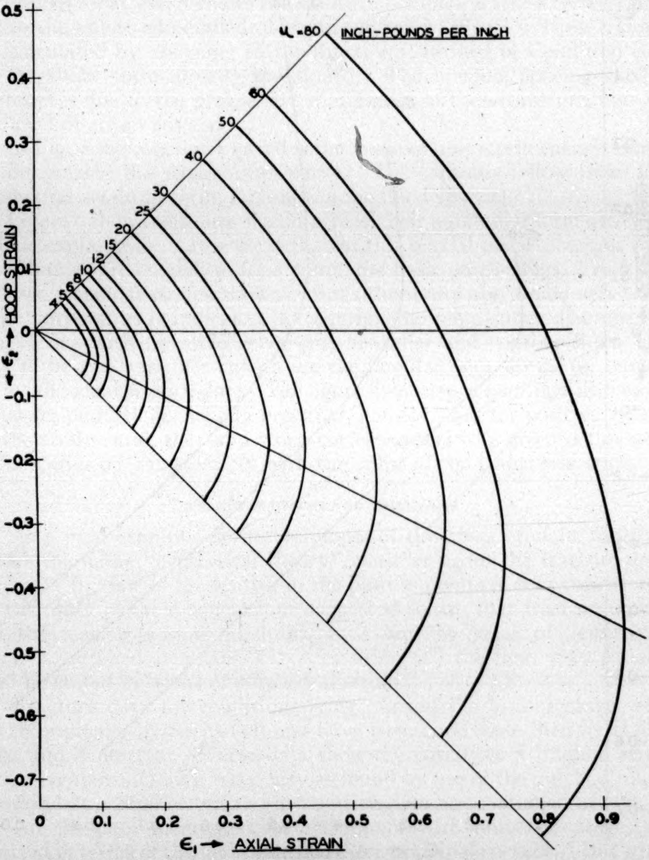


FIG. 5. Contours of energy (in inch-pounds per cubic inch) of plastic deformation (under biaxial stress, $\sigma_3 = 0$).

horizontally and the relation between the two ratios m and n is expressed by the curved line. When instead of n, m is laid off horizontally as well as vertically, it results in the straight diagonal line also drawn in.

The subsequent figures reproduce the co-ordinates in the left side of

Fig. 2 on a doubled scale, and show the contours of various functions. Thus Fig. 3 shows the relation between octahedral strain and its components, axial and transverse. Of course there is also another transverse strain component not zero (except at $m = -1$), but it is not necessary to show this component since its value can always be found

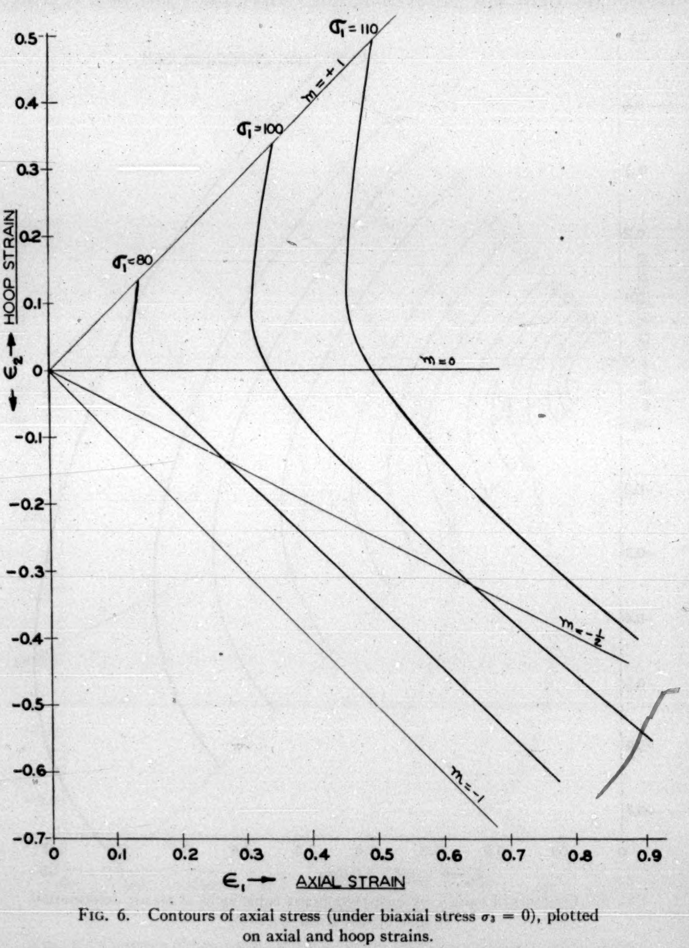


FIG. 6. Contours of axial stress (under biaxial stress $\sigma_3 = 0$), plotted on axial and hoop strains.

if needed from the basic Condition (*B*). Figure 3 performs the same service for strain as the lower octant of Fig. 1 for stress.

In the same way, Fig. 4 does for strain what the upper octant of Fig. 1 does for stress; it shows the values of octahedral stress corresponding to the values of octahedral strain plotted in Fig. 3. These values are calculated by assigning to the function identified in Condition (*C*) the parabolic form already mentioned. The unequal spacing of the contours is due to the progressive diminution of the secant modulus as the level of strain advances.

In Fig. 5 the function plotted is the value of unit strain energy found by integrating the stress-strain curve. The contours follow those for octahedral strain in form, but show a modified spacing.

Figure 6 shows contours of axial stress; their unfamiliar form deserves a little explanation. It is clear that in this biaxial phase diagram two co-ordinates are enough to fix a point; the basic pair in Figs. 2 to 6 are the two principal strains shown. But other pairs also would serve the same purpose, as is indicated by covering the rectangular diagram for the strains with an alternative grid of radial and contour lines. In Fig. 6 the two variables chosen are the familiar ones, strain (or stress) ratio, and axial stress value. The figure illustrates a fact for which most of us are perhaps not wholly prepared, namely, that for positive values of the strain ratio, the axial stress corresponding to a given octahedral value varies only moderately with the value of the transverse stress.

FRACTURE STRESS AND TRIAXIALITY

As a final example of the usefulness of the ideas brought together under the name "octahedral theory" consider again the fracture data in (**14**). In view of the scatter in the plotted points it is as true as any of the other possible conclusions suggested to say that fracture occurs (in the tension-tension quadrant) at a limiting value of "effective" (that is, octahedral) stress. By Condition (*C*) the limit may equally well be stated in terms of octahedral strain.

Fracture data for conditions lying outside the biaxial range were given originally by Sachs (**15**) and have been cited since then by Hollomon and Zener (**16**). These data show the variation of fracture stress (axial component) with triaxiality as found by use of the notched round tension bar. Similar but more extensive data are contained in (**17**).

All the data mentioned, in accordance with universal usage, are plotted in terms of the highest of the three principal stresses. But when the algebraic relationships of octahedral theory, or graphic relations like those shown in Fig. 6, are used for converting the data to terms of octahedral stress, it is found that fracture occurs at a value of octahedral stress or strain that is at least roughly constant. When fracture is associated with a limiting value of octahedral stress, the effect of triaxiality is thus completely accounted for.

If, until it is disproved, we may accept this hypothesis, an important simplification in thought about flow and fracture of metals will have been made.

REFERENCES

(1) WENDELL P. ROOP, "Stress and Strain in Plastic Flow," *Welding Journal*, September, 1946.

(2) D. C. DRUCKER, "A Reconsideration of Deformation Theories of Plasticity," Paper No. 48-A-81 presented to the ASME, December 2, 1948.

(3) D. C. DRUCKER, "The Plasticity of Metals. A Discussion of Mathematical Theory and its Application to Civil Engineering Problems," paper presented at the Annual Meeting of the ASCE, January, 1949.

(4) GEORGE SACHS AND J. D. LUBAHN, "Bending of an Ideal Plastic Metal," paper presented to the ASME, December 2, 1948.

(5) WM. M. BALDWIN, JR., "Graphical Methods of Representing Some Conditions of Plasticity," AIMME, TP 1980, *Metals Technology*, April, 1946.

(6) A. NADAI, "Plasticity, a Mechanics of the Plastic State of Matter," New York, McGraw-Hill Book Co., 1931, Chapter 14.

(7) S. J. FRAENKEL, "Experimental Studies of Biaxially Stressed Mild Steel in the Plastic Range," Paper No. 48-APM-1, presented to the ASME, June, 1948.

(8) F. R. SHANLEY, "Plastic Theory in Sheet Metal Forming Problems," *Jour. Aeronautical Sciences*, Vol. 9, p. 313 (1942). See also this author's book "Basic Structures," New York, John Wiley & Sons, Inc., 1944.

(9) WM. R. OSGOOD, "Plastic Bending," *Jour. Aeronautical Sciences*, Vol. 11, p. 213 (1944).

(10) J. A. VAN DEN BROEK, "Theory of Limit Design," New York, John Wiley & Sons, Inc., 1947.

(11) G. SACHS, J. D. LUBAHN AND D. P. TRACY, "Drawing Thin-walled Tubing with a Moving Mandrel Through a Single Stationary Die," *Jour. Applied Mechanics*, (1944).

(12) G. SACHS AND L. J. KLINGER, "The Flow of Metals Through Tools of Circular Contour," *Ibid.*, Vol. 14, p. 88 (1947).

(13) E. SIEBEL, "Die Formgebung im Bildsamen Zustande"; translated by John H. Hitchcock and published serially in *Steel*, October, 1933 to May, 1934.

(14) JOHN E. DORN, "Effect of Stress State on the Fracture Strength of Metals," in "Fracturing of Metals," Cleveland, Am. Soc. for Metals, 1948.

(15) GEORGE SACHS, J. D. LUBAHN AND L. J. EBERT, "The Effects of Notches of Varying Depth on the Strength of Heat Treated Low Alloy Steels," *Transactions*, Am. Soc. for Metals, Vol. 34, p. 517 (1945). See especially Fig. 15.

(16) J. H. HOLLOMON AND C. ZENER, "Problems in Fracture of Metals," *Jour. Applied Physics*, Vol. 17, p. 82 (1946). See Fig. 7, also, in J. H. HOLLOMON, "Flow and Fracture of Metals under Combined Stress," paper presented to ASME, December 2, 1948.

(17) GEORGE SACHS, "Mechanical Relations in Fracturing of Metals," an unpublished report to the Office of Naval Research, May, 1947.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS*

VERY THIN CRYSTAL OSCILLATOR PLATES

The increasing interest in high frequencies for radio communication is accompanied by a demand for very thin quartz crystal oscillator plates having fundamental frequencies up to 100 megacycles or higher. In crystals whose frequency is in the higher range, the thickness of the quartz plate determines the frequency; the higher the frequency the thinner the crystal must be. A crystal with a fundamental frequency of 100 Mc is about 0.001 in. thick, and its surfaces must be parallel within a few millionths of an inch.

The usual crystal grinding methods and machinery have proven inadequate for producing plates of the required thickness. The National Bureau of Standards has therefore developed improved equipment, capable of producing 0.001-in. thick quartz crystals with a high degree of parallelism and flatness. The apparatus can also be used to produce equally thin wafers from a variety of other materials, for example, extremely thin dielectric plates for miniature radio condensers.

Ordinarily, crystals are carried in a planetary path between two abrasive-charged lapping plates by a thin apertured disk called a nest. Nests thinner than 0.005 in. do not have the strength required to carry the crystals. Because the nest must be thinner than the crystals to permit their abrasion, crystals produced by this method have maximum fundamental frequencies of about 20 Mc.

The initial problem, therefore, was to make the crystal thickness independent of the nest thickness. The solution involved various replacements for the customary top lapping plate and related changes in the design of the nest.

The apparatus developed is an improved form of the square block and cell method and exists in two slightly different models—the inkwell type and the tall plunger type. The inkwell type has a conical exterior and is essentially a keyed and closely fitting plunger and cylinder. The crystal is attached to the plunger by means of a drop of oil; the unit is then inverted and placed on the lapping plate. The crystal is thus confined between the piston and plate by the cylinder walls. A nest drives a number of such units over the lapping plate. The tall plunger model differs mainly in having a taller piston sliding on bearing screws by which the amount of wobble can be precisely controlled.

TOTAL OZONE RELATED TO TROPOSPHERE TEMPERATURES

Measurements of the ultraviolet radiation reaching the earth from the sun have revealed an important correlation between the temperature of the air at an altitude of about five miles and the total amount of ozone in the atmosphere. The ozone concentration was measured at the National Bureau of Standards in Washington, D. C., and compared with weather and sunspot data from the U. S. Weather Bureau and other sources. Observations were made on a total of 244 days during the nine-year period, 1934 to 1943.

---

* Communicated by the Director.

The radiation was measured with titanium phototubes sensitive to radiant energy in the wavelength region between 3000 and 3400 Å., and with glass filters whose transmittance increases with wavelength. Ozone readily absorbs ultraviolet energy and any change in the relative amount of ozone in the atmosphere is accompanied by a change in the transmittance of the glass filters attached to the phototube.

The ozone concentration varies with the seasons from a minimum in late summer to a maximum in late winter, and has long been believed to have a direct relationship with the weather itself. It appears impossible, however, to establish any definite connection between pressure patterns and ozone value, but there is a direct relationship between air movements above 4 or 5 miles and ozone. The seasonal variation of ozone may be explained by temperature changes at the boundary layer (tropopause) between the low-lying troposphere and the stratosphere. The layer rises in the spring and drops again in the fall.

In the troposphere, air movement by convection tends to take place vertically, while in the stratosphere the air is stratified into horizontal layers and there is little vertical circulation. Very little ozone normally exists within the troposphere, because air currents rapidly bring about conditions of pressure and temperature that accelerate its degeneration. On the other hand, ozone within the upper ten per cent of the ozone layer rapidly comes to equilibrium as the result of normal photochemical reactions.

The experimental results show very little ozone below the tropopause. Temperature irregularities in the lower part of the atmosphere would therefore not be expected to affect the amount of ozone directly. But when a disturbance occurs at a much higher level, resulting in a change in the effective position of the tropopause, fluctuations in the ozone value would be expected.

PHOTO-GRID TECHNIQUE FOR MEASURING SHEET-METAL ELONGATION

Since processes which improve the strength of sheet metal frequently reduce the ductility, the National Bureau of Standards has recently developed an improved photo-grid technique for determining elongation of sheet metal. In addition, the new technique is proving useful in the investigation of plastic deformation in the vicinity of holes and in studies of other structural discontinuities which result in excessive stresses. This marked advance in the technique of measuring elongation over short gage lengths followed the development of the photo-grid process for ruling gage marks on test specimens.

A master grid was prepared at the Bureau by ruling the grid in wax on plate glass, etching the lines in the glass, and filling them with lead sulfide. These lines are about 0.015 mm. wide and are spaced nominally 0.25 mm. apart. In reproducing the master grid lines on the test specimen, lines of excellent quality were obtained with the photoengraver's product known as cold top enamel. The thoroughly cleaned specimen is mounted on a whirler and coated with a small amount of cold top enamel. The whirler is then run at about 500 rpm. for 10 min. or until the enamel on the strip is dry. The sensitized specimen is removed from the whirler, printed in contact with the film negative of the grid in a vacuum frame, and developed by immersion in agitated cold top developer. A dye in the developer makes the image on the metal visible. After developing, a length equal to 50 or 100 spaces near the middle of the gage length of the specimen is measured to determine the average grid spacing.

PERFORATED COVER PLATES FOR STEEL COLUMNS

A study of the mechanical properties of perforated steel cover plates for bridge columns, utilizing full-scale models, has been completed by the National Bureau of Standards. Most large compression members of bridge trusses have a box-type cross section in order to provide efficient utilization of the material in resisting column action. It is desirable to provide openings in the member to allow access to the interior for inspection and painting; common practice is to use two channels with a cover plate on one side and lattice bracing or battens on the other. The lattice bracing or battening serves to brace the flanges of the channels or angles and to transfer shear, but it does not contribute to the cross-sectional area effective in resisting thrust. Recently a more economical design in which the bracing is replaced by a perforated cover plate has been used. The purpose of the investigation was to determine the effects of the perforations on the structural properties of these plates.

The cover plates were tested for stress distribution, strength, and axial stiffness. Tests in the elastic range were made on columns built up from plates having circular, ovaloid, elliptical, and square perforations, as well as on columns with solid plates for control. The axial stiffness of the columns was measured in a 600,000-lb. capacity testing machine.

This work indicates that perforated cover plates contribute to the strength, and especially to the stiffness, of columns built up from the plates and angles. Formulas have been derived that may be applied whenever data are required concerning displacements in a structure having perforated members, and the validity of these formulas has been confirmed by experiment. At the same time it has been found that the net area of a perforated-plate column may safely be used for design purposes even for a perforation shape that produces great stress concentration.

PERFORMANCE OF A BASEBOARD HEATER

The use of baseboard heating systems in low cost housing has increased in recent months, particularly in basementless houses where a first-floor heating system is necessary. In order to acquire adequate technical data on systems of this type, the National Bureau of Standards included an investigation of a typical baseboard heating system as part of its program on heat transfer phenomena. The tests covered the temperature distribution in a four-room test bungalow under various outside temperature conditions, in addition to the installation and operation problems involved.

Several types of recently developed baseboard heating systems use copper, steel, or cast-iron elements not much thicker or higher than a baseboard around the entire perimeter of the living space. The system installed in the test bungalow consisted of lengths of finned $1\frac{1}{4}$-in. iron pipe connected by 1-in. standard pipe, and partially enclosed by a perforated sheet-metal cover.

A conventional wall-type electric thermostat, a modulating control employing an outside thermostat, and a three-way valve that mixed the boiler supply and return water to vary the temperature of the water entering the system were used for the tests.

The Bureau's test bungalow, consisting of four rooms and a bath, is completely enclosed by an insulated outer shell so that the temperature and humidity may be varied to simulate any condition throughout the United States. Temperatures were measured in the test bungalow by means of thermocouples placed at various distances above the floor, and on the exterior walls, attic, and floors.

The temperature distribution was observed in the bungalow while a temperature of 70° F. was maintained for outside air temperatures between 50° and 0° F. The test results showed that comparatively small temperature differences existed between rooms and between different levels in the same room with this type of heating system. The maximum temperature difference between any two rooms in the living zone (the space between 2 and 60 in. above the floor) was 1.4° F. with the interior doors open and 3.6° F. with the interior doors closed.

The Bureau's investigation showed that the specimen baseboard convector system produced smaller vertical temperature differences in the test bungalow than any other system previously studied. The baseboard heating system is well suited for application to basementless houses because it is installed above the floor level and because it provides better comfort than most other systems at the floor level. However, this type system can also be used in houses with basements where the boiler is located below the floor.

THE FRANKLIN INSTITUTE

MINUTES OF THE STATED MEETING

February 17, 1950

The Stated Monthly Meeting for February was postponed from Wednesday, the fifteenth, until Friday, the seventeenth, at which time the second lecture of the Edward G. Budd Memorial Lectureship was presented.

The meeting opened at 8:15 P.M. in the Lecture Hall with a capacity house in attendance.

The President invited comment or correction to the minutes of the January meeting which appeared in full in the February issue of the JOURNAL OF THE FRANKLIN INSTITUTE. Since no changes were recommended, the minutes were accepted as printed.

The following increases in the membership of the Institute were announced:

Sustaining	7
Active	47
Associate	25
Student	33

At the end of January, the total membership was 5953.

Mr. William L. Batt, the speaker of the evening, was introduced by Mr. Nalle. Mr. Batt, President of S.K.F. Industries, presented a challenging and stimulating lecture entitled "Vision Knows No Boundaries," as the second of the Edward G. Budd Lectures on management problems. This talk will be published in full in a subsequent issue of the JOURNAL.

HENRY B. ALLEN,
Secretary

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in the JOURNAL within the next few months:

LEITNER, A. AND R. D. SPENCE: The Oblate Spheroidal Wave Functions.

FO-VAN CHANG: Trigonometric Series Applied to the Bending of Long Rectangular Plates to a Cylindrical Surface.

HEPBURN, JOSEPH S.: The Influence of the Temperature and the Period of Keeping upon the Biochemical Changes in the Common Fowl *Gallus domesticus*.

HERSHBERGER, W. D. AND L. E. NORTON: Servo Theory Applied to Frequency Stabilization with Spectral Lines.

MICHAELSON, HERBERT B.: Variations of Grid Contact Potential and Associated Grid Currents.

HEREFORD, FRANK L.: Concerning Meson Produced Secondary Electrons Emerging from Thin Foils.

PENNER, S. S.: Application of Near-Equilibrium Criteria During Adiabatic Flow to Representative Systems.

THORNTON, WILLIAM M., JR. AND EDWARD S. HAUBER: Zirconium as a Material for Fractional Weights.

BEYER, ROBERT T. AND MING-YI WEI: An Extension of a Theory of Magnetic Amplifiers.

MUSEUM

THE ORUKTER AMPHIBOLIS

One of the exhibits most commonly borrowed for display outside the Museum is a model of a nightmarish construction called the Orukter Amphibolis. This odd looking piece of mechanism was the first automobile produced in this country, and it was built almost a full century before the commonly accepted birth date (1892) of the gasoline automobile. In addition, it was an amphibian, the prototype of the Army "duck." The vehicle has all the essential parts of an automobile, body, wheels, axles, and engine. The fuel differed from that used today, for the engine, being steam driven, was fed with wood. While primitive, it operated successfully.

The strange amphibian was constructed in 1805 by Oliver Evans, who was America's most famous millwright and who rivals Richard Trevithick as being the first to use the expansive force of steam for practical purposes. In 1772, when he was seventeen years old, Evans became interested in the problem of propelling carriages and wagons by some means other than by animal power. He rejected as impracticable the attempts previously made in which the passenger propelled the vehicle by means of treadles, ratchet wheels, cranks, and such like. A local blacksmith's boy made an experiment which gave Evans a suggestion for development. The touchhole of a gun barrel was stopped up, water poured into the barrel, and firmly wadded. The gun breech was then inserted into the smithy fire. Presently, the wad was discharged with, to the boy, a gratifying report. On learning of this experiment, Evans perceived the possible use of the force employed.

Eventually he developed a non-condensing engine in which the power was derived from the expansion of high pressure steam. In 1786 he petitioned the Legislature of the State of Pennsylvania for the exclusive right to run his proposed steam wagons on the roads of that State. The Legislature did not act upon his petition but the State of Maryland granted him the required right for a period of fourteen years. However, Evans could not persuade anyone to invest capital in the enterprise and the appearance of his power propelled land vehicle was postponed.

In 1804 he received a contract to build a dredging barge to the order of the Philadelphia Board of Health. He installed his own steam engine to operate the chain of buckets and then was seized with an inspiration. "When the work was finished I put wheels under it, and though it was equal in weight to two hundred barrels of flour and the wheels were fixed on wooden axletrees for this temporary purpose and in a very rough manner, and attended with very great friction of course, yet with this small engine I transported my great burthen to the Schuylkill with ease; and when it was launched into the water, I fixed a paddle wheel at the stern and drove it down the Schuylkill to the Delaware and up the Delaware to the city, leaving all the vessels going up behind me at least half-way."

Evans does not here refer to a spectacle presented by his strange craft when the first axles broke under the great weight. His workmen, who had greater faith in him than had the capitalists, replaced the deficient axles at their own expense. The Orukter was then operated around Center Square, now Penn Square, for several days, and spectators were invited to contribute 25 cents each to defray the cost of the new axles. Thus Philadelphia held its first automobile show.

The Museum's model was built and presented by Mr. Greville Bathe, the biographer of Evans.

THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

***Abstract of* Mechanical Computing Mechanisms.**[1]—ROBERT R. REID[2] AND DURAY E. STROMBACK.[2] This paper consisted of a series of four articles on the design and development of mechanical analog computers. The first article explained the differences between digital and analog computation, and discussed typical analog quantities. Design specifications were listed along with a statement of the variable relation in equation or tabular form, a knowledge of the range of input values and suitable sensing elements or input devices. The output of the computer may indicate, record, or control another mechanism. Schematic design began with block diagrams of various methods of solution. Various types of errors in computers were discussed, and methods of calibration and adjustment were illustrated. Figure 1 shows nine types of calibrating devices for computing mechanisms.

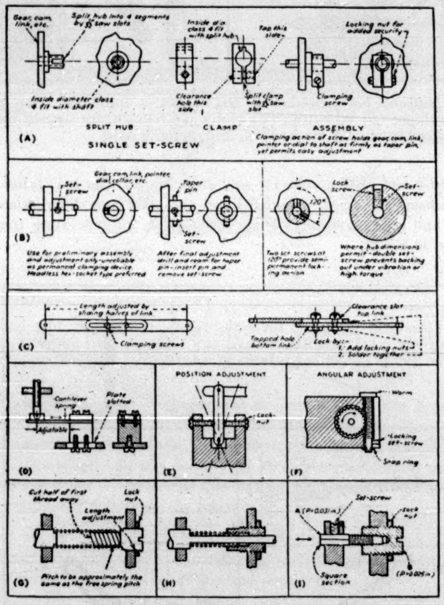

Gear, cam, link, etc.
Split hub into 4 segments by $\frac{1}{32}$ saw slots
Inside diameter class 4 fit with shaft
Inside dia class 4 fit with split hub
Tap this side
Clearance hole this side
Split clamp with $\frac{1}{32}$ saw slot
Locking nut for added security
Clamping screw
SPLIT HUB
CLAMP
ASSEMBLY
(A)
SINGLE SET-SCREW
Clamping action of screw holds gear, cam, link, pointer or dial to shaft as firmly as taper pin, yet permits easy adjustment
Set-screw
Gear, cam, link, pointer, dial, collar, etc.
Taper pin
Set-screw
120°
Lock screw
Set-screw
(B)
Use for preliminary assembly and adjustment only-unreliable as permanent clamping device. Headless hex-socket type preferred
After final adjustment drill and ream for taper pin-insert pin and remove set-screw
Two set screws at 120° provide semi-permanent locking action
Where hub dimensions permit - double set-screw prevents backing out under vibration or high torque
Length adjusted by sliding halves of link
Clamping screws
(C)
Clearance slot top link
Tapped hole bottom link
Lock by:-
1. Add locking nuts
2. Solder together
Cantilever spring
Plate slotted
Adjustable
(D)
POSITION ADJUSTMENT
Lock-nut
(E)
ANGULAR ADJUSTMENT
Worm
Locking set-screw
Snap ring
(F)
Cut half of first thread away
Lock nut
Length adjustment
Pitch to be approximately the same as the free spring pitch
(G)
(H)
Set-screw
A (P=0.031 in.)
Lock nut
Square section
(P=0.025 in.)
(I)

FIG. 1. Nine types of calibrating devices for computing mechanisms are shown: (*A*) split hub and clamp; (*B*) set screw arrangements; (*C*) adjustable link length; (*D*) cantilever spring rate adjustment; (*E*) link positioner; (*F*) angular adjustment; (*G*) helical spring rate adjustment; (*H*) another helical spring adjustment; (*I*) differential screws.

The second article began the analysis of individual mechanisms for function generation, including trigonometric functions. Cam mechanisms were of chief concern in function generation, although certain other devices are useful in computing trigonometric functions, for

---

[1] Published in *Product Engineering*, August through November issues, 1949.

[2] Research Engineers, The Franklin Institute Laboratories for Research and Development, Philadelphia 3, Pa.

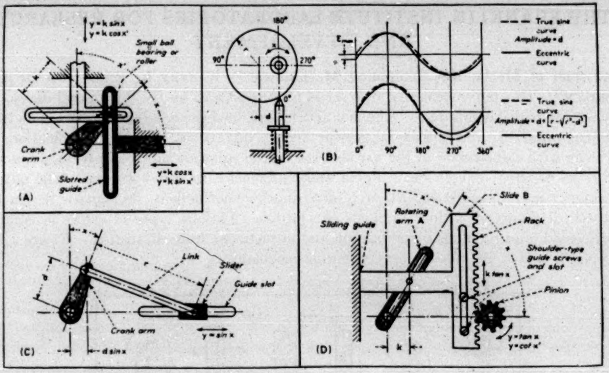

y = k sin x
y = k cos x'
Small ball bearing or roller
Crank arm
Slotted guide
y = k cos x
y = k sin x'
(A)
180°
90°
270°
0°
(B)
True sine curve Amplitude = d
Eccentric curve
True sine curve Amplitude = d + [r − √(r² − d²)]
Eccentric curve
0° 90° 180° 270° 360°
Link
Slider
Guide slot
Crank arm
y = sin x
d sin x
(C)
Slide B
Rotating arm A
Sliding guide
Rack
Shoulder-type guide screws and slot
k tan x
Pinion
y = tan x
y = cot x'
k
(D)

FIG. 2. Mechanisms for reproducing trigonometric functions: (*A*) Scotch yoke for sine-cosine relationships; (*B*) eccentric; (*C*) crank mechanism, for approximating sine-cosine functions; (*D*) tangent-cotangent mechanism, approximate only.

example the eccentric and Scotch yoke for sine and cosine. The four types of addition and subtraction mechanisms described, included bevel and spur gear differentials, link and slide types, and a rack and gear variation. These mechanisms for generating trigonometric functions are illustrated in Fig. 2.

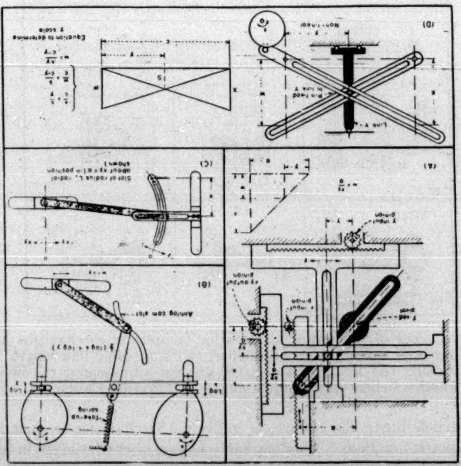

(A)
(B)
(C)
(D)
Equation to determine x scale
Non-linear
Pin fixed to slide
Line of
Slot of radius L, radial about xy = 0 (in position shown)
Analog cam slot
Take-up spring
x input pinion
y output pinion
Fixed pin

FIG. 3. Multiplication mechanism based on different principles: (*A*) similar triangles multiplier; (*B*) logarithmic multiplier; (*C*) slide-link multiplier, approximate only; (*D*) slide multiplier having one non-linear scale.

The third article dealt with devices for accomplishing multiplication and division, integration and differentiation. Methods using similar triangle relations, and logarithmic addition were given, supplemented with two approximate linkage methods for multiplication. Division is more difficult because of the need for dealing with conditions at or near zero. A method of servo-division was explained. The ball and disc integrater was explained, together with methods for zero shift. Reversing a ball-disc integrater provided a differentiating device. A time-rate differentiating mechanism uses viscous fluid forces. Methods of temperature compensation were presented. The mechanisms pictured in Fig. 3 are several variations of multipliers. These devices may be modified to perform division.

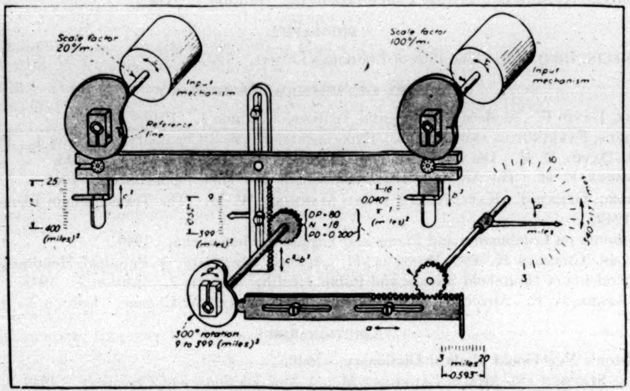


FIG. 4. Final schematic design of the proposed computer is shown, with scale factors. This design, based on the cam method, was selected because far fewer parts (and potential mechanical errors) are included. The value of careful analysis is demonstrated.

The fourth article discussed the synthesis of components into a workable computer design, taking as example the Pythagorean relation concerning the right triangle. By block diagrams and schematic design analysis, two rival computers were brought to the end of the engineering calculation stage. At this point it can be shown what factors will cause acceptance of one design over the other. The device schematically represented in Fig. 4 is one of several possible analog computers for solving the relation

$$a = \sqrt{c^2 - b^2}.$$

The series concluded with a short discussion of such design considerations as limit stops, electrical components and effects of vibration and acceleration.

LIBRARY

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. The average cost for a print 9 × 14 inches is thirty-five cents.

The Library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A.M. until 5 P.M.; Wednesdays and Thursdays from 2 P.M. until 10 P.M.

RECENT ADDITIONS

ARCHITECTURE AND BUILDING

Adlam, Thomas Napier. Radiant Heating. Edition 2. 1949.

Severns, William Harrison and Fellows, Julian R. Heating, Ventilating and Air Conditioning Fundamentals. Edition 2. 1949.

Starbuck, Robert Macy. Practice and Theory of Radiant Heating. 1949.

AVIATION

Abbott, Ira H. and Van Doenhoff, Albert E. Theory of Wing Sections. 1949.

Norwood, Arthur G. Private Pilot's Handbook. Edition 2. 1949.

BIOGRAPHY

Franklin, Benjamin. The Pith of Franklin's Letters. 1927.

CHEMISTRY AND CHEMICAL TECHNOLOGY

Boltz, David F. Modern Instrumental Analysis. Volume 1. 1949.

Daniels, Farrington and Others. Experimental Physical Chemistry. Edition 4. 1949.

Fierz-David, H. E. Die Entwicklungsgeschichte der Chemie eine Studie. 1945.

Saunders, K. H. The Aromatic Diazo-compounds and Their Applications. 1949.

Seaborg, Glenn T.; Katz, Joseph J. and Manning, W. M. The Transuranium Elements. 1949.

Symposium on Combustion and Flame and Explosion Phenomena. 1949.

Tinkler, Charles K. and Masters, H. Applied Chemistry; a Practical Handbook for Students of Household Science and Public Health. Volume 2. Edition 2. 1948.

Van Arkel, A. E. Moleculen en Kristallen in de Anorganische Chemie. Edition 3. 1949.

DICTIONARIES

Blakiston's New Gould Medical Dictionary. 1949.

Miall, Stephen and Miall, Laurence M. A New Dictionary of Chemistry. 1949.

ELECTRICITY AND ELECTRICAL ENGINEERING

Blum, William and Hogaboom, George B. Principles of Electroplating and Electroforming (Electrotyping). 1949.

Starling, Sydney G. Electricity in the Service of Man. 1949.

ENGINEERING

Templin, Richard Laurence and Others. Properties of Metals and Materials Engineering. 1949.

GEOLOGY

Brooks, G. E. P. Climate Through the Ages. 1949.

Fritsch, Volker. Grundzuge der angewandten Geoelektrik. 1949.

Kendrew, W. G. Climatology. 1949.

Linsley, Roy K. Jr.; Kohler, M. A. and Paulhus, Joseph L. H. Applied Hydrology. 1949.

HISTORY

Eavenson, Howard Nicholas. Map Maker and Indian Traders. 1949.

MANUFACTURE

Simmons, Richard F. Custom Built Rifles. 1949.

Treloar, L. R. G. The Physics of Rubber Elasticity. 1949.

West, Trustham F.; Strausz, H. J. and Barton, D. H. R. Synthetic Perfumes. 1949.

MATHEMATICS

Synge, John L. and Schild, A. Tensor Calculus. 1949.

MECHANICAL ENGINEERING

Addison, Herbert. Centrifugal and Other Rotodynamic Pumps. 1948.
Colvin, Fred Herbert and Stanley, Frank A. Grinding Practice. 1950.

METALLURGY

International Conference on the Physics of Metals. 1949.

PHOTOGRAPHY

Johnson, S. C. Foulsham's Complete Photographer. 1949.

PHYSICS

Dusinberre, G. M. Numerical Analysis of Heat Flow. 1949.
Garlick, G. F. J. Luminescent Materials. 1949.
Ower, E. The Measurement of Air Flow. Edition 3. 1949.
Purday, H. F. P. An Introduction to the Mechanics of Viscous Flow. 1949.
Ruheman, M. The Separation of Gases. Edition 2. 1949.
Sommerfeld, Arnold. Mechanik. Edition 4. 1949.
Tyler, F. Intermediate Heat. 1949.

PLASTICS

Howard, R. N. Strength of Plastic and Glass. 1949.

RADIO

Smith, R. A. Aerial for Metre and Decimetre Wave Lengths. 1949.

TEXTILES

The Fibro Manual. 1949.
Mauersberger, Herbert R. Rayon Looms and Weaving. No Date.

MEMBERSHIP

ACTIVE MEMBERS ELECTED AT THE MEETING OF THE BOARD OF MANAGERS, FEBRUARY 15, 1950

ACTIVE FAMILY

Norman G. Altman
Donald D. Hayes
Frederick W. Slack
John S. Williams
William V. Zambelli

ACTIVE

Edward F. Allen
Samuel Belber
Harold Boericke, Jr.
William E. Boettner, Jr.
Elmer Frank Borrelli
Walter Bracken
W. S. Bradford
Charles M. Cooper
Percival Coulter
Louis H. Dworkin
Fred W. Flessa
Victor P. Head
W. F. Henn
Fred J. Jankuskas
Martin W. Jeiven
Henry B. Jones, Jr.
Samuel C. Jones
John W. Klopp, M.D.
John Markle, 2nd
Fred G. Medinger, M.D.
Bert Miller
Mrs. John Rogers
Milton D. Rosenau
Harold W. Scott
Lee F. Sierp
Miss Anna K. Stimson
John E. Stringer
N. L. Whitecotton
Howard C. Wickes
C. Y. Wilgus
James P. Zallea

ACTIVE NON-RESIDENT

Vice Adm. Harold G. Bowen
Theodore C. Cuyler, 3rd
Martin S. Maier, Ph.D.
Vivian Eberle Spencer, Ph.D.

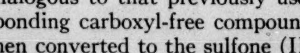
NECROLOGY

Anthony A. Hausser '28
Harrison Hoblitzelle '43
Jacob D. Lit '36
Louis W. Wheelock '36
W. D. Wiggins '36
Maurice L. Wurzel '42

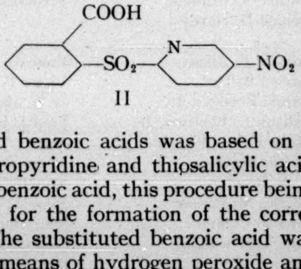
NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

The Preparation of Several Substituted Phenyl 2-Pyridyl Sulfides and Sulfones.—FRANCIS E. REINHART. In a previous publication (**5**) from this laboratory the preparation of a number of substituted phenyl 2-pyridyl sulfides and sulfones, some of which were tested for antistreptococcal activity in mice, was described. Although no activity in this respect had been observed, it has since been noted that one of the compounds, 5-nitro-2-pyridyl phenyl sulfone (I), completely inhibits the growth of *Staphylococcus aureus* in broth culture at a concentration of 12 mg. per 100 ml. This concentration represents approximately the limit of solubility of the substance in aqueous media.

In view of the activity shown by this compound, it was considered of interest to prepare several related compounds which would possess greater water solubility and which might show increased bacteriostatic properties. It has been reported (**3**, **4**) that two related sulfones, 2-(*p*-aminophenylsulfonyl)-5-nitropyridine and 2-(*p*-aminophenylsulfonyl)-5-aminopyridine, possess antistreptococcal activity which can probably be ascribed to their close chemical relationship to the highly active *p*,*p*′-diaminodiphenyl sulfone. The present report concerns the preparation and testing of *o*-(5-nitro-2-pyridylsulfonyl)-benzoic acid (II), the corresponding amino and acetamido compounds, and the three corresponding sulfides. Both (II) and the corresponding sulfide have previously been reported (**1**) but since alternate methods of preparation were used in this laboratory their description has been included.



I



II

The preparation of the substituted benzoic acids was based on a fusion reaction between 2-chloro-5-nitropyridine and thiosalicylic acid to yield *o*-(5-nitro-2-pyridylmercapto)-benzoic acid, this procedure being analogous to that previously used (**5**) for the formation of the corresponding carboxyl-free compound. The substituted benzoic acid was then converted to the sulfone (II) by means of hydrogen peroxide and to *o*-(5-amino-2-pyridylmercapto)-benzoic acid by reduction with stannous chloride. Acetylation of this amino compound followed by oxidation with potassium permanganate gave *o*-(5-acetamido-2-pyridylsulfonyl)-benzoic acid which, in turn, yielded the corresponding free amino compound on hydrolysis with hydrochloric acid.

These compounds were tested for bacteriostatic activity on several strains of *S. aureus* and the results are recorded in Table I. Under the conditions employed, it is seen that *o*-(5-nitro-2-pyridyl-mercapto)-benzoic acid was the only one which displayed inhibitory activity. The corresponding sulfone, which chemically more nearly resembles 5-nitro-2-pyridyl phenyl sulfone, was found to be inactive. Tests on this compound, however, are considered to be of doubtful significance, since independent experiments showed that the substance slowly decomposes in neutral, aqueous solution.

o-(5-Nitro-2-pyridylmercapto)-benzoic Acid.—A mixture of 2-chloro-5-nitropyridine (15.9 g.) and thiosalicylic acid (17 g.) was pulverized in a mortar and then heated for $1\frac{1}{4}$ hr. in a flask which was immersed in an oil bath at 100°–110° C. Hydrogen chloride was evolved. The solid reaction product was washed with water and then dissolved in 10 per cent sodium hydroxide solution (100 ml.). After the addition of water (300 ml.), treatment with dilute hydrochloric acid precipitated the yellow sulfide which was filtered off and washed with water. Recrystallization from glacial acetic acid (360 ml.) gave 25.2 g. of the pure product.

TABLE I. *Properties and Analyses of Compounds*

Benzoic Acid	M.p., °C. (uncor.)	Yield, %	Analyses, %				Minimal[b] Inhibitory Conc., mg./100 ml.
			Nitrogen		Sulfur		
			Calc.	Found	Calc.	Found	
o-(5-Nitro-2-pyridylmercapto)-[a]	206–8	91			11.61	11.79	50
o-(5-Nitro-2-pyridylsulfonyl)-[a]	171–2	62			10.40	10.48	inactive
o-(5-Amino-2-pyridylmercapto)-	204–6 dec.	78	11.38	11.29	13.02	12.95	inactive
o-(5-Amino-2-pyridylsulfonyl)-	237–9 dec.	86	10.07	10.00	11.52	11.58	inactive
o-(5-Acetamido-2-pyridylmercapto)-	204–5 dec.	80	9.72	9.52	11.12	11.22	inactive
o-(5-Acetamido-2-pyridylsulfonyl)-	226–7 dec.	78	8.75	8.68	10.01	10.13	inactive

[a] Previously described (1).

[b] The chemically defined medium employed for growth of staphylococcus by McIlwain (2), with addition of nicotinamide ($10^{-6}M$), was used; compounds were added to the medium as solutions of their sodium salts to give final graded concentrations up to 150 mg. per 100 ml. Solutions were sterilized by filtration and observations made 24 hr. after inoculation with *S. aureus*. Incubation was carried out at 37° C.

The preparation of this acid from diazotized anthranilic acid and 5-nitro-2-pyridinethiol has been reported by Colonna (1).

o-(5-Nitro-2-pyridylsulfonyl)-benzoic Acid.—Pulverized *o*-(5-nitro-2-pyridylmercapto)-benzoic acid (8 g.) was suspended in glacial acetic acid (80 ml.). After the addition of 30 per cent hydrogen peroxide (14 ml.) the mixture was kept at room temperature for 48 hr. with occasional shaking. The crude sulfone which formed was separated by filtration and washed with water. Recrystallization from 200 ml. dilute ethanol (50 per cent by volume) gave 4.1 g. of colorless plates. An additional amount (1.5 g.) was obtained as a precipitate when the reaction mixture filtrate, after standing 48 hr., was diluted with an equal volume of water.

Chromic acid was used by Colonna (**1**) for the preparation of this compound from the sulfide.

o-(5-Amino-2-pyridylmercapto)-benzoic Acid.—To a suspension of *o*-(5-nitro-2-pyridylmercapto)-benzoic acid (20 g.) in concentrated hydrochloric acid (140 ml.) was added with stirring during $\frac{1}{2}$ hr. a solution of stannous chloride dihydrate (54 g.) in concentrated hydrochloric acid (90 ml.). It was necessary to cool the reaction mixture to maintain the temperature at 40–50° C. After being kept for an additional hour at this temperature the mixture was cooled and then filtered to obtain the insoluble stannic chloride-amine complex which had formed. The latter was decomposed in a mixture of water (500 ml.) and concentrated hydrochloric acid (65 ml.) to give a precipitate of the hydrochloride of the desired amine. After several hours this precipitate was removed by filtration and washed with a little 2 *N* hydrochloric acid. Traces of tin were removed by dissolving it in *N* sodium hydroxide (250 ml.), adjusting the pH to 7.0–7.5 with dilute hydrochloric acid, and filtering off the slight amount of precipitate with the aid of Celite. Careful addition of dilute hydrochloric acid to the filtrate (pH, 3–5) precipitated orange colored prisms of the desired amino compound which were separated by filtration and washed with water. Purification was effected by solution in water (250 ml.) containing 2 *N* sodium hydroxide solution (65 ml.), treatment of this solution with Darco at 50° C. for 1 hr., and reprecipitation with dilute hydrochloric acid at a pH of 3–5. The product (14 g.) was obtained as colorless needles. For analysis, 6 g. were recrystallized from 95 per cent ethanol (370 ml.) to give 4 g. of small, colorless prisms. In 2 *N* hydrochloric acid these are converted to insoluble needles which appear to be the hydrochloride.

o-(5-Amino-2-pyridylsulfonyl)-benzoic Acid.—A suspension of *o*-(5-acetamido-2-pyridylsulfonyl)-benzoic acid (6 g.) in 6 *N* hydrochloric acid (60 ml.) was heated to 100° C. for $1\frac{1}{2}$ hr. On cooling, a mass of fine needles (probably the hydrochloride of the desired compound) formed. Addition of water (120 ml.) caused the formation of the free amino derivative which was separated by filtration and washed with water. Colorless prisms (4.5 g.) were obtained by recrystallization from 450 ml. dilute ethanol (30 vol. per cent); these were soluble in warm ethanol and practically insoluble in water.

o-(5-Acetamido-2-pyridylmercapto)-benzoic Acid.—Ten grams of *o*-(5-amino-2-pyridylmercapto)-benzoic acid were dissolved in water (150 ml.) to which 2 *N* sodium hydroxide (25 ml.) had been added. After further dilution with water (150 ml.) the solution was brought to incipient precipitation with a small amount of 2 *N* hydrochloric acid. The solution was then cooled in an ice bath while acetic anhydride (6 ml.) was added with stirring during $\frac{1}{2}$ hr. At the same time 2 *N* sodium hydroxide solution was introduced at a rate such that precipitation of the amine and its acetylated derivative was prevented. Stirring was continued for 1 hr., after which time the product was precipitated by the addition of acetic acid, removed by filtration and washed with cold water. With the aid of Darco, recrystallization from 900 ml. dilute ethanol (25 vol. per cent) yielded 9.4 g. small colorless prisms. The compound dissolved in warm alcohol but not in water.

o-(5-Acetamido-2-pyridylsulfonyl)-benzoic Acid.—Eight grams of *o*-(5-acetamido-2-pyridylmercapto)-benzoic acid were dissolved in *N*/5 sodium hydroxide (200 ml.) and this solution was treated with dilute sulfuric acid to the point of incipient precipitation. It was then stirred at 40° C. while potassium permanganate (6.3 g.) dissolved in water (150 ml.) was gradually added during 1 hr. After the mixture had been stirred for two more hours the manganese dioxide was removed by filtration and washed with water made slightly alkaline with sodium hydroxide. The filtrate, combined with the washings, was adjusted to pH 5–6 and treated with Darco at 50° C. After removal of the latter, the product was precipitated by adding dilute sulfuric acid to pH 2. Colorless prisms (6.9 g.) were obtained on filtration and washing with water. The compound was recrystallized from 95 per cent ethanol as colorless flat needles.

REFERENCES

(1) M. Colonna, *Boll. sci. facoltà chim. ind., Bologna*, p. 145 (1941): *Chem. Abs.*, **37**, 4399 (1943).

(2) H. McIlwain, *Brit. J. Exp. Path.*, **21**, 136 (1940).

(3) F. Nitti and J. Matti, *Compt. rend. soc. biol.*, **136**, 401 (1942).

(4) R. O. Roblin, Jr., J. H. Williams and G. W. Anderson, *J. Am. Chem. Soc.*, **63**, 1930 (1941).

(5) H. C. Winter and F. E. Reinhart, *J. Am. Chem. Soc.*, **62**, 3508 (1940).

BOOK REVIEWS

Albert Einstein: Philosopher and Scientist, edited by Paul Arthur Schilpp. 781 pages, port., 16 × 24 cm. Evanston, The Library of Living Philosophers, 1949. Price, $8.50.

The monumental "Library of Living Philosophers" has been enriched by the addition of this volume dealing with the man who possesses the most profound mind of our generation. The work is the tribute of the leading contemporary scientists to the great philosopher-scientist, honoring him on the seventieth birthday. The contributors, twenty-five in number, include Arnold Sommerfeld, Niels Bohr, Max Born, Louis de Broglie, Max von Laue, and Wolfgang Pauli. Each contributes a weighty study of some aspect of, or approach to, Einstein's work. Because the contributors are eminent in their own right they do not hesitate to disagree with some of Einstein's opinions and they produce some formidable equations to support their contentions. Nor do these writers fail to ask searching questions about his meaning. However, the subject of their homage has the last word, for the editor had the happy thought of permitting Einstein to reply to his critics at the end of the book.

The most significant essay in the volume is that contributed by Einstein himself, and which he calls "Autobiographical Notes." Very little is known of the personal story of the great man whose modesty has been a shield against the curious. Anyone who turns to this essay in the hope of learning something of his private life will be wasting time. It is an intellectual autobiography, telling what he thinks and how he arrived at his conclusions. An interesting feature of the "Notes" is the account of the scientific theories in vogue at the time when Einstein began his studies. The milestones of his youth are recalled as the first time he saw a compass and his introduction to the works of Euclid. Both incidents set in operation a series of thoughts which demonstrate beyond all doubt that Einstein was a most unusually precocious child. By the age of sixteen he had acquired a command of differential and integral calculus, and with this knowledge he began to explore "this huge world, which exists independently of us human beings and which stands before us like a great eternal riddle.". Taking nothing for granted he submitted everything that was offered to his mind to a rigid examination.

Although the theory of relativity is traced from its inception in the author's mind while he was still a youth, the reader will find no simple account of its development. There never was anything simple about it. Results did not fall into Einstein's lap; every stage of the investigations was attained and passed only through prolonged hard work. As a result of the reading of these "Notes" we are reminded that Einstein contributed more to contemporary physics than the theory of relativity. He did remarkable work on the Brownian movement, on statistical thermodynamics, and equilibrium fluctuations. Nor should it be overlooked that his conception of light quanta started physicists in their search for the means of synthetizing the wave and the corpuscular theories of light. This work alone would have entitled him to a foremost place among modern physicists had not these achievements been overshadowed by his epoch-making work on the theory of relativity. In every investigation he undertook Einstein first mastered the problems and then gave to them a novel aspect which had escaped other investigators. His elaboration of the transformation formulas of Lorentz is a notable example of this faculty.

Coming hard on the heels of the announcement that he has developed a field theory which integrates gravitation and electromagnetism this volume serves a useful purpose in surveying the grounds of preparation for the new theory much more completely than can be found elsewhere. The other contributors to the volume provide authoritative commentaries upon various aspects of Einstein's work, but nowhere is it light reading, and an extensive knowledge of mathematics is presupposed. The book is so complete that it is likely to remain the standard work on the great philosopher in the light of contemporary knowledge.

T. C.

Giant Brains or Machines That Think, by Edmund Callis Berkeley. 270 pages, illustrations, 15 × 22 cm. New York, John Wiley & Sons, Inc., 1949. Price, $4.00.

"Giant Brains or Machines That Think"! Can machines really think? Just what is thinking? The author answers both of these questions in his opening chapters and then goes on to describe several different large scale computers. The detailed descriptions are limited to machines that existed in 1948, but the author does provide brief descriptions of machines now being constructed.

The layman and the scientist or engineer in other fields will enjoy this book. Also it provides an extensive bibliography for anyone who wants to delve further into the field of computing machinery. Mr. Berkeley has gone to great pains to write a popular and easily understood book about a complicated subject. Like some of our picture magazines, he too believes one picture is worth a thousand words, and he has profusely illustrated his book with diagrams and drawings.

The description of an elementary relay computer having all the essentials of a large digital computer may entice some of his readers to build one of these devices. A science museum or college might well use one for illustrative purposes. Mr. Berkeley lovingly calls this brain child "Simple Simon."

Two intriguing mathematical concepts that will be new to many readers outside the field of mathematics and computing machinery are explained. The first concept is the binary system of numbers. The existence and usefulness of a counting system other than our everyday decimal system comes as a surprise to many people. This system is explained in a mathematical appendix.

The other subject is logical or Boolean algebra, in place of words. Boolean algebra uses letters and a well-defined set of symbols to determine the truth of a set of statements. For two or three statements it is more of interest than of practical value, but as the number of conditions to be tested increases, logical algebra can clarify and simplify our thinking tremendously. The selection of life insurance risks, the writing of contracts and the designing of calculating machinery are all places where logical algebra now is used. All those conflicting campaign promises made by politicians would show up as false statements in a quick and short analysis on the logical truth calculator.

The main subject matter of the book is its descriptions of large computing machinery. This important science has blossomed rapidly in the last five years and has captured the imagination of many scientists. Mr. Berkeley describes the design and use of the commerical punched card machine systems, the electromechanical differential analyzer of M.I.T., the relay computers of the Bell Telephone Company and Harvard University and the Eniac, an electronic computer. This variety is supplemented by a description of the Kalin-Burkhart Logical Truth Calculator. The variety of design principles is well illustrated in this survey of existing machines.

The author is much concerned with the future design and application of large computers. He foresees some wonderful things. He also sees the possibility of some terrible consequences if the machines are not used properly.

This book has caused considerable discussion and it makes provocative and interesting reading.

E. A. MECHLER

TRILINEAR CHART OF NUCLEAR SPECIES, by William H. Sullivan. 14 × 28 cm. New York, John Wiley & Sons, Inc., 1949. Price, $2.50.

This chart presents in very compact manner a great deal of information concerning the nuclear properties of most of the known isotopes. It employs a trilinear coordinate system giving a simultaneous plot of A *versus* Z, $A-Z$ *versus* Z, and $A-Z$ *versus* A. A represents the mass number and Z the atomic number. This is arranged in such a way that nuclides having a constant value of A (isobars) lie along a vertical line. All those having a constant value of Z (isotopes) lie along a line inclined 30° above the horizontal, while all those having a constant value of $A-Z$ (isotones) lie along a line inclined at 30° below the horizontal. The nuclides themselves are represented by close fitting hexagons on the chart. A color code is used to indicate whether the nuclide occurs in nature or is produced artificially. Colors are also used to indicate whether or not beta or alpha stability is associated with the particular nuclide in question. Symbols within the hexagons give additional information concerning: relative abundance of the natural isotopes, atomic mass values, nuclear spin, slow neutron cross section, half-life and several properties of the nucleus.

This attractive and colorful chart should prove to be of much value to scientists interested in nuclear research and to teachers of courses involving nuclear physics, since it presents such a large amount of useful information in a graphic and easily accessible manner.

MARSHALL D. EARLE

THEORY OF WING SECTIONS, by Ira H. Abbott and Albert E. von Doenhoff. 693 pages, illustrations and tables, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1949. Price, $15.00.

Here is a welcome addition to the library of the rapidly-expanding field of the aeronautical sciences. This book should serve as an extremely handy and useful reference for the practicing aeronautical design engineer and theoretical aerodynamicist alike. The bulk of the material presented has been covered previously in numerous classified and unclassified NACA (National Advisory Committee for Aeronautics) publications of past and recent years; however, since the authors had participated in the major part of this NACA research, they were therefore in an excellent position to consolidate and compile in this one volume only those data on the aerodynamics of wings at subcritical speeds considered the most important and useful. In view of the present trend toward specialization and the many, many scattered publications on the subject heretofor readily available only to a few, the authors have made a valuable contribution by screening and compiling this information for the numerous engineers and aerodynamicists who must keep up-to-date on a wide variety of subjects.

Alternative theoretical approaches to the subject and many experimental data have been rigorously excluded in order to keep the book at a reasonable length, and these exclusions prevent any claim to complete coverage of the subject. In the theoretical developments the authors have taken care to state the assumptions and to review briefly the elementary principles involved. Additionally, they have attempted to keep the mathematics as simple as is consistent with the difficulties of the problems treated; however, a knowledge of differential and integral calculus and of elementary mechanics is presupposed.

The text itself contains only 307 pages including 191 illustrations and a listing of 167 references. The Theoretical treatment progresses from elementary considerations to methods used for the design of NACA low-drag airfoils. Methods and data are presented for using wing-section data to predict wing characteristics, and judiciously selected plots and cross-plots of experimental data are presented for readily useful correlation, justification or non-justification of certain simplifying assumptions made in the theoretical analyses.

The general headings contained in the text include: the significance of wing-section characteristics, simple two-dimensional flows, theory of wing sections of finite thickness, theory of thin wing sections, the effects of viscosity, families of wing sections, experimental characteristics of wing sections, high-lift devices, and the effects of compressibility at subsonic speeds.

The four appendices cover 378 pages and present comprehensively the basic thickness forms, mean lines, airfoil ordinates, and experimental aerodynamic characteristics on all the many NACA wing sections including the very latest laminar-flow, low-drag sections.

SAMUEL M. BERKOWITZ

THE AIRPLANE AND ITS ENGINE, C. H. Chatfield, C. Fayette Taylor, and Shatswell Ober. Fifth Edition, 380 pages, illustrations, 16 × 23 cm. New York, McGraw-Hill Book Co., Inc., 1949. Price, $4.50.

The authors bring up-to-date a fine text book issued last in 1940. Commentary and illustrations (all new) of aircraft, engines and accessories have been included and modified to keep abreast of the latest developments in jet airplanes and jet propulsion. As in prior editions' the basic treatment is non-mathematical; however, equations dealing with theory are presented in simple basic terms sufficient for computation of the less complicated characteristics of airplane flight and propulsion. Text material covers, in an easy-to-understand manner, the various fundamentals of the airplane including its parts, lift and drag, stability and control, reciprocating and jet engines, propulsion and the propeller, performance, maneuvers, structure, instruments, and types of airplanes.

R. H. McCLARREN

BASIC THEORIES OF PHYSICS; MECHANICS AND ELECTRODYNAMICS, by Peter Gabriel Bergmann. 280 pages, illustrations, 14 × 22 cm. New York, Prentice-Hall, 1949. Price, $3.75.

Professor Bergmann has set himself the task of preparing a critical introduction to theoretical physics. The author has led his reader through some of the difficulties and inconsistencies which faced the founders of classical physics. In accomplishing this task he has presented in a concise manner many of the tools necessary for the study of modern physics.

Consider two charged particles moving at given velocities along given paths. Applying Lorentz's force equation and adding the two resulting force expressions, one gets the net rate of change of the total linear momentum of an isolated system of two moving point charges. This expression is not in general zero and it expresses the deviation from Newton's third law of the interaction of the two charges. In addition, the equations are such that it can readily be seen that the magnetic force excited by one particle on the other is not the same in any frame of reference. Maxwell's theory of the electromagnetic field effectively restored the conservation laws. Action at a distance laws were not sufficient to cope with electromagnetic phenomena. The special theory of relativity in turn made the equations of electrodynamics obey transformation laws which were in keeping with experimental evidence.

The foregoing outline is fairly typical of the book's approach to the development of classical mechanics and electrodynamics. The nature of the book is such that the problems are an extremely important part of the material. This is not only because their solution demands careful thought, but also because so much material is covered in fairly scant detail. Such subjects as tensor calculus and the properties of guided waves cannot be adequately discussed in a few pages.

It seems unfortunate that the author used the electrostatic and Gaussian system of units rather than the prevalent rationalized MKS system. A treatment of thermodynamics, statistical mechanics, and quantum mechanics will be presented in a forthcoming companion volume.

E. S. KRENDEL

FACSIMILE, by Charles R. Jones. 422 pages, illustrations, 15 × 24 cm. New York, Murray Hill Books, 1949. Price, $6.00.

This book is a well-written, rather thorough volume on the subject of facsimile. The book is composed of four major parts entitled "What Facsimile Is," "How Facsimile Works," "Present-Day Facsimile Systems," and "Servicing."

The first part covers both the history of facsimile's development and the uses made of facsimile today. The second part is of a technical or semi-technical nature discussing all of the major components of facsimile transmitting and reproducing devices and accessory devices and including a chapter on standards. The third part considers the electrical, physical, and operational characteristics of each. The fourth part takes up general maintenance of facsimile equipment and detailed maintenance of several particular equipments.

This book includes numerous photographs of facsimile equipments and components in addition to many reproductions of pictures which have been transmitted by facsimile. Circuit diagrams of a number of equipments are also shown and discussed.

"Facsimile" is hardly a text of interest to the designer of facsimile equipment, since it does not treat in very great detail the underlying theory of data transmission. However, the book will be of value to the user or to maintenance personnel, and it is to this audience that it appears to have been directed.

ROBERT S. GRUBMEYER

THIRD SYMPOSIUM ON COMBUSTION FLAME AND EXPLOSION PHENOMENA, published under the auspices of the Standing Committee on Combustion Symposia. 748 pages, illustrations, 18 × 26 cm. Baltimore, The Williams and Wilkins Co., 1949. Price, $13.50.

This book is a compilation of papers and discussions presented at the Third Symposium on Combustion, Flame and Explosion Phenomena, held at the University of Wisconsin, September 7-11, 1948.

The preface contains a brief history of the two previous combustion symposia held in the United States and the activities leading up to the third symposium. Then, following the table of contents and the list of authors, an introduction to the material presented is given by Bernard Lewis, the Chairman of the Committee for the Third Symposium. He explains the division of the field of combustion research into three main divisions: (1) chemistry and kinetics of slow and explosive oxidation reactions; (2) ignition and propagation of flame; (3) state of the burned gas. The enlargment of scope in these three fields is mentioned and the development of combustion research, during the past decade, in the specific fields covered by the papers presented in the book is briefly outlined.

The book is divided into the following phases of combustion research, the number following each division indicating the number of papers presented: (*a*) flame stabilization and quenching (10); (*b*) flame propagation in explosive gas mixtures (19); (*c*) flames of fuel jets (3); (*d*) ignition of gas mixtures (8); (*e*) kinetics and mechanism of combustion reactions (21); (*f*) flame spectroscopy and radiation (6); (*g*) burning and detonation of explosives (9); (*h*) thermodynamics of flame gases and thermochemistry (7); (*i*) experimental techniques (7); (*j*) combustion in engines and rockets (6); and (*k*) gas burners and furnaces (4).

Due to the international character of the meeting, the book represents a summary, through 1947, of the knowledge in the field of combustion research. It is not a text book, but is an invaluable reference book to research workers. Contributions were made by practically every noted worker in the field, and the papers presented, plus the references and discussion following the papers, represent the nearest approach possible to the presentation of all the available data concerning combustion research. Anyone engaged in research of any type which is related in any way to the problems encountered in combustion research should not be without this volume in his or her personal library.

JOHN T. AGNEW

NUMERICAL ANALYSIS OF HEAT FLOW, by G. M. Dusinberre. 227 pages, illustrations, 15 × 24 cm. New York, McGraw-Hill Book Co., 1949. Price, $4.50.

Anyone confronted with the task of determining the heat flux and temperature distribution in a solid body or a composite structure under steady state or transient conditions should welcome this book as a valuable aid. Written for the "practical man," it presents in an elementary manner the technique of Southwell's "relaxation method" as applied to problems of heat transport by conduction, surface transfer, convection and radiation encountered in engineering practice.

With the exception of eight pages containing definitions of the prerequisite physical concepts, five pages on which several simple solutions are derived analytically, and four pages given to miscellaneous observations on methods of attack, Fourier's work, "quantum theory and relativity" and convergence criteria, about seventy-five pages of text each are devoted to steady-state and transient problems in "constant-properties" systems, arranged in the order of increasing complexity of their numerical analysis. Systems with variable thermal properties are considered on additional twenty-one pages. Numerous problems (with answers) to be solved by the reader are appended to each chapter.

The author rightly stresses the power of numerical methods in finding practically significant solutions of problems whose analytical solution is hopelessly complex or altogether impossible at present. Although but the simplest arithmetic operations of addition, subtraction, multiplication and division are used at each stage of the process of solution, the number of individual operations increases rapidly with the number of reference points used, and may become prohibitively great. The author is careful to point out the importance of the proper choice of the number and location of the reference points for obtaining a solution of sufficient accuracy with the least amount of computational labor. This reviewer found the pages dealing with irregular networks of reference points and the sections devoted to transient-flow problems especially interesting.

It is regrettable that the quality of the presentation of the material is not on par with its contents. In the list of symbols, the dimensions of a number of physical quantities are not given. The letter R is used to denote both "thermal resistance" and "residual heat flow." The letter ρ (lb. per cu. ft.) is said to denote "density" rather than "specific weight." Many definitions are worded rather carelessly and are apt to confuse the reader not familiar with the subject. For example (p. 5), in an anisotropic material the *conductivity* (*not* the conductance) varies with the orientation with respect to the axes of structural symmetry at each point of the material (*not* with respect to the "axes of the system"). Hence, k is *not* a function of the coordinates in a homogeneous anisotropic material, contrary to the statement in the text. This reviewer feels that the value of this useful book would be enhanced if its text were freed from blemishes of the above-mentioned types.

O. R. SPIES

SOUND ABSORBING MATERIALS, by C. Zwikker and C. W. Kosten. 174 pages, illustrations, 16 × 25 cm. New York, Elsevier Publishing Co., 1949. Price, $3.00.

There is a wealth of books on the acoustics of sound absorbing materials, the basis of these publications being empirical in nature, listing the acoustic properties of various materials as found experimentally and propounding methods of design based upon the analysis of empirical data. These methods of sound absorbing material design were the only ones used until recently, when materials began to appear which were designed by theoretical analysis of the physical principles of the wave mechanics involved. This book fills the need for a text on designing by theoretical analysis.

A sound knowledge of the mathematics of wave mechanics and, of course, representation in the complex plane is a prerequisite, even though the book makes some attempt at a review of wave mechanics. It is not an engineers' handbook in any sense, but is a text on the fundamental theory of sound absorption. The book is a typical text on theory, in that it treats the problems using the assumptions of all sound waves being normal to the material surface and thermal and viscous effects being negligible. The theory which is treated is largely an extension of Kirchoff's analysis of wave mechanics in absorbing media.

The book derives in detail the theory of wave propagation as affected by impedance and absorption in homogeneous media, in cylindrical tubes with extension to porous media and in elastic media. A good feature is that a large portion of the book is given to methods of experimentally measuring these factors and evaluating the theory. Methods of measuring absorption and impedance are discussed. The two final chapters are devoted to the practical aspects of absorption by resonators and under non-ideal conditions of oblique and homogeneous angles of incidence. Throughout the book are analogies to the well developed theories of electrical wave mechanics.

The authors are not particularly up-to-date in their knowledge of new sound absorbing materials, using only one example. However, the book will be useful as a timely text to those in the field of acoustics who wish to go beyond the general "engineering" concepts of sound absorbing material.

A. F. CARY

ELECTRONICS IN ENGINEERING, by W. Ryland Hill. 274 pages, illustrations, 16 × 24 cm. New York, McGraw-Hill Book Co., 1949. Price, $3.50.

"Electronics in Engineering" is a text intended primarily for engineering students other than those whose major field is electrical or electronics engineering. The book is a very well balanced combination of theory and practical application, covering basic emission theory, vacuum tubes, amplifiers (both timed and untimed), oscillators, rectifiers, and modulation and demodulation. There are also chapters on photosensitive devices, control circuits, the cathode ray oscilloscope and the vacuum tube voltmeter, and transducers.

The author has attempted, wherever practicable, to discuss mechanical and other analogies to electronic circuits. This is not only helpful to the engineering student, but is also thought-provoking material for the engineer with design problems.

The book is well illustrated with diagrams and circuits. The chapters are accompanied with an ample supply of illustrative examples as well as with problems for the student to work.

It is a bit discouraging to note the lack of bibliographical references, since it is believed that such material would be very useful to the audience for whom the text is intended. Although references would be a welcome addition to the text, their absence does not detract from the thorough and practical manner of presentation employed by the author.

ROBERT S. GRUBMEYER

ENGINEERS' DICTIONARY: SPANISH-ENGLISH AND ENGLISH-SPANISH, by Louis A. Robb. Second edition, 664 pages, 14 × 22 cm. New York, John Wiley & Sons, 1949. Price, $12.50.

One of the important war-time contributions to the necessary industrial and commercial cooperation between North and South America was the publication of Louis A. Robb's "Engineers' Dictionary: Spanish-English and English-Spanish," with its wealth of in-use engineering terms. The second edition of the "Engineers' Dictionary," just published, follows the general plan of the first one, with these changes: the number of electrical and mechanical engineering terms is greatly increased; a complete list of radio terms is included; and the important words for the shipbuilding, logging, sugar milling, and oil drilling industries are now listed. The emphasis is naturally on civil engineering, the author's specialty, but the wealth of material indicates that the author has consulted with experts in all fields of engineering.

As evidence of the additional material that has been added in this second edition, it is interesting to note that the number of entries for "drill" has been increased from 22 to 34, and for "perno" there are now 81 items instead of the 51 which were given in the first edition. Also, the gender of Spanish nouns is now indicated.

The book is designed for quick and easy use, for there are almost no cross-references to confuse the picture. In general, each word group is listed only once, under the first word in the group: for example, "internal-expanding clutch" appears alphabetically in the *i*'s and nowhere else; conversely, in the Spanish-English section, such a phrase as "bajo calor de endurecimiento" appears only under "bajo." This device saves a considerable amount of page-thumbing, thus permitting a faster translation than could ever be obtained with the cross-reference type of dictionary, where it is often necessary to look up three or four words before finding the desired combination.

"Engineers' Dictionary" is the most practical answer to the technical language barrier that it has been my privilege to examine. The terms are those in actual use by the engineers in the countries involved; the pages are not cluttered with solid columns of type, for each new phrase is set on a new line, indented under the lead word; the type is quite legible; and the single-entry system greatly simplifies the actual looking-up of terms.

The book is strongly recommended not only for engineers, but also for translators.

NANCY S. GLENN

FOURIER METHODS, by Philip Franklin. 289 pages, illustrations, 14 × 21 cm. New York, McGraw-Hill Book Co., 1949. Price, $4.00.

This new introductory book to Fourier series and Laplace transforms was written for engineering and physics students and technologists who require a basic knowledge of harmonic analysis, complex exponentials, Fourier integrals, Fourier transforms and Laplace transforms. The author broadly interprets Fourier methods as any analysis or synthesis of functions by a linear process applied to sines, cosines, or to complex exponentials. These wide limits necessitate that many of the subjects be treated in bare essentials, with emphasis on basic information.

An introductory chapter, devoted to complex quantities, developes the hyperbolic functions and the elementary concepts pertaining to the complex plane. In the second chapter Fourier series and integrals are introduced. It is felt that the first eighty pages of text could have been materially reduced with no loss by omitting the detailed development of special rules for integrating components of periodic functions.

Commencing with Chapter 3, the author presents in turn, material pertaining to partial differential equations, boundary value problems, and Laplace transforms and transients. Partial differential equations which arise in elementary problems of heat flow, vibrations, electric transmission lines, and electro magnetic radiation are developed and solved for their general solutions and for their particular solutions in special cases. Needless to say, the solutions of interest are those which can be expressed by Fourier methods. In setting up the equations, emphasis has been placed on an explanation of the physics of the situation, an approach many times overlooked in texts on applied mathematics. Essentially the same physical problems are considered in the chapter devoted to boundary value problems where solutions of the partial differential equations consistent with assigned boundary conditions are investigated. The last chapter, devoted to the Laplace transform, introduces the subject in the conventional manner, first by defining the transform, then applying the transformation to a system of ordinary differential equations, this followed by illustrations of the procedure by elementary problems on the lossless long transmission line.

The author has successfully treated the subject of Fourier series and transforms and Laplace transforms in an introductory manner, which while not sufficiently comprehensive for a serious student of the subject, is adequate for the many practical problems that may be encountered by engineers who do not require the more detailed treatment accorded the subject in other texts. The thirty-one sets of problems, answers to which are provided, consider applications which supplement the text to the extent which leads this reviewer to remark "If you plan to read the book, plan to solve the problems," for they are an essential part of the text.

S. CHARP

THE EXTRAPOLATION, INTERPOLATION AND SMOOTHING OF STATIONARY TIME SERIES WITH ENGINEERING APPLICATIONS, by Norbert Wiener. 163 pages, 15 × 24 cm. New York, John Wiley & Sons, Inc., 1949. Price, $4.00.

The published version of this book is identical to that issued as a classified report during the war years. The theory set forth is widely used and has guided modern thinking in filter and predictor theory.

In the introduction and in Chapter I, the author summarizes certain mathematical and engineering concepts which are useful in the text proper. Chapter II discusses the prediction problem for a stationary time series. Here it is shown that the auto-correlation coefficient of the series is the only statistical parameter required for minimizing a mean square prediction error (an optimum linear method). Certain technical difficulties are also discussed. The filter problem for a single time series, treated in Chapter III, is discussed in a similar manner. The linear predictor and filter for multiple time series is considered in Chapter IV. The concluding chapter (V) discusses certain miscellaneous problems encompassed by the techniques set forth in the previous chapters.

Two appendices by Dr. Norman Levensin are welcome additions to this book. The first discusses the Wiener root mean square error criterion in filter design and prediction while the second gives an exposition of Wiener mathematical theory of prediction and filtering.

A. D. HESTENES

THE CHARACTERISTICS OF ELECTRICAL DISCHARGES IN MAGNETIC FIELDS, edited by A. Guthrie and R. K. Wakerling. 376 pages, illustrations, diagrams, 16 × 24 cm. New York, The McGraw-Hill Book Co., 1949. Price, $3.50.

The National Nuclear Energy Series, of which this book is Volume 5 of Division I (Electromagnetic Separation Project) activities, is a record of the scientific and technical contributions of the various Manhattan Project Technical Sections. In this particular case, the University of California Radiation Laboratory was the center of research on the characteristics of electrical discharges in magnetic fields, with emphasis on the cases of discharges in the vapors of uranium compounds. The subject volume is essentially a collection of especially edited reports originally written for use on the Manhattan Project, with but minor changes from the original reports to satisfy rules for declassification of data of military value.

Electronic physicists and engineers interested in the subject of fundamental phenomena will be delighted with the wealth of theoretical and experimental data. In a subject so new and so pressing at the time it is no small wonder that the Radiation Laboratory was interested primarily in experimental devices for studying arc phenomena, and their relationship to the various diffusion processes for the uranium compounds.

Most of the eleven chapters in the book describe in detail experiments and results of experiments, many of which were new and never before reported in the literature. The introductory chapter, providing a unifying framework for the rest of the book, develops a general qualitative theory for the internal state of an arc in a magnetic field of strength to 10,000 gauss, although most experimental work was done at about 3000 gauss. This is followed by a chapter on the experimental use of probes for plasma exploration, modifying the theory of the Langmuir-type probe to make it applicable to the arc in a magnetic field. The idea of following theory by experiment is carried out in the discussion pertaining to the required minimum pressure for stable arc operation. The pressure limitation is found necessary to supply sufficient ions to the filament to satisfy the Langmuir condition at the double sheath. Good agreement with theory is obtained for the cases of argon, helium, and sulfur hexafluoride. Similar agreement with newly developed theory is obtained in the experiments to measure the absolute values of cross-section for ionization of uranium tetrafluoride, and the ionization and dissociation of uranium tetrafluoride and uranium hexafluoride by homogeneous electron impact.

Detailed descriptions of experiments, and apparatus and the results of experiments with changing conditions of pressure, magnetic field, arc conditions, geometry, and other factors close the book.

While not intended as a text book, this volume, offering so much to the basic scientist, should be useful as a professional reference text. While references to the literature are few, this is quite understandable in view of the pioneering work which is described.

S. CHARP

BOOK NOTES

HEATING, VENTILATING AND AIR-CONDITIONING FUNDAMENTALS, by William H. Severns and Julian R. Fellows. Second edition, 666 pages, illustrations, 16 × 24 cm. New York, John Wiley & Sons, 1949. Price, $6.50.

The time lapse since the first edition of this book in 1937 has necessitated rather extensive revision, for there have been considerable changes in techniques and equipment during the period. Panel heating and all-year air-conditioning have been treated at length. Many new illustrations and problems have been substituted for out-dated ones.

ASTRONOMY AND NAVIGATION, by D. H. Sadler. Occasional Notes No. 13, Royal Astronomical Society, 109 to 126 pages, illustrations, plates, 17 × 25 cm. London, Royal Astronomical Society, 1949. Price, 3s. 6d.

This issue of the *Occasional Notes* of the Royal Astronomical Society deals with the recent progress in the techniques and application of astronomical navigation, emphasizing the interdependence of astronomy and navigation. A brief historical review is given, followed by an account of the relationship of the Royal Astronomical Society with navigation. Modern developments are considered, particularly the changes which have been made in nautical almanacs both for surface and air use. The paper concludes with some interesting speculations on possible developments of the future.

INTRODUCTORY COLLEGE CHEMISTRY, by Joseph A. Babor and Alexander Lehrman. Second edition, 814 pages, illustrations, plates, 15 × 24 cm. New York, Thomas Y. Crowell Co., 1949. Price, $4.75.

This textbook is designed for courses where the students have had no high-school chemistry or where no account is taken of such preparation. The same authors' "General College Chemistry" is planned for students with a background of high-school chemistry. In this new edition such topics as nuclear transformations and their applications, silicates and their structure, high polymers, fluorine compounds, and ion exchangers have been expanded.

16-MM SOUND MOTION PICTURES, A MANUAL FOR THE PROFESSIONAL AND THE AMATEUR, by William H. Offenhauser, Jr. 580 pages, illustrations, diagrams, 15 × 23 cm. New York, Interscience Publishers, Inc., 1949. Price, $10.00.

"Within the last two decades the term 16-mm has become something more than the mere designation of a film width; it is now almost symbolic of all non-theatrical films." With these words Mr. Offenhauser introduces his book and suggests the importance of 16-mm film in the world of today. As a training medium during the last war it played an important role and its use for industrial purposes has been increasing rapidly. In view of its importance and the fact that most of the pertinent literature is to be found in periodicals, the necessity of a well-rounded work on 16-mm film is apparent. Such is the work in hand.

Associated with the industry for more than twenty years, Mr. Offenhauser has made many contributions to its development and is widely conversant with its problems. He has relied heavily on the *Journal of the Society of Motion Picture Engineers* in producing this work.

After a brief discussion of the history of 16-mm film and its relation to other types, the author gives a brief summary of the different factors which go into the making of a 16-mm picture. With the exception of the script, the various factors are discussed at length in the subsequent chapters.

Beginning with the film itself, Mr. Offenhauser considers both its physical and sensitometric characteristics including the various types of film—original, duplicating and release print. A chapter considers dimensions and standards in 16-mm, while another considers the problem of 16-mm emulsion position.

Different types of cameras and camera equipment are briefly described. Then follow two lengthy chapters on sound recording considering the basic problems involved, the characteristics of different systems, the description of various types of equipment and the best methods of operation.

After the original film has been made there are the problems of editing and assembly, processing and release printing, preservation and storage, all of which are separately treated. After the film is ready, there is still the need of securing the most favorable conditions for its release and the twin topics of picture and sound projection as they affect the audience are considered. Various industrial applications of 16-mm sound motion pictures are outlined as is also the relation between 16-mm film and television. Appendices include a glossary and other useful data.

The book is generously illustrated and forms a comprehensive treatise on 16-mm film which should prove useful to a wide group of readers.

YEAR BOOK OF THE TEXTILE INSTITUTE NO. 2 1949–50. 258 pages, illustrations, 15 × 24 cm Manchester, The Textile Institute, 1949. Price, 10/.

In addition to information about the Textile Institute and a list of its members, this volume contains other useful material. There is a directory of more than two hundred textile organizations, primarily English, but including a few from other countries. A bibliography on textile literature includes sections on introductory literature, literature for students, and literature currently available.

Ten standards relating to textiles have been included as well as a section on standardized terms and definitions. The need for a universal system of yarn counts is discussed and comparative tables of yarn equivalents are given.

CURRENT TOPICS

Push-Button Farm Boy.—Johnny Clive Williams, a farm boy from Fayette, Ala., is being hailed as a 16-yr. old combination of Burbank, Bromfield and Edison. His achievements in putting electricity to work on the farm have won him a Westinghouse scholarship award as one of six national winners in the 4-H Better Methods Electric Contest sponsored by the Westinghouse Educational Foundation, which is maintained by the Westinghouse Electric Corporation.

Believing that tomorrow's farmer should do more button-pushing and less back-bending, Johnny has put into practice on the farm what he has learned at 4-H Club meetings. As a result, he has:

1. Built an automatic device to solve the problem of counting and dusting with DDT powder 30,000 gladioli bulbs each season. The ingenious machine, powered by electricity, does a 24-hr. job in about 3½ hr. (see below).
2. Converted a hot-plate heating unit, an old lard can, and a metal cover into an electric chick brooder which has meant healthier chicks and fewer lost each year.
3. Rewired the farm house, wired the barn, built such equipment as electric fans and lamps, and repaired motors.
4. Applied the latest farm "know-how" to win 19 prize ribbons with his own registered dairy cattle.
5. Practiced modern cultivation techniques to get higher yields from his corn crops.

A junior in Fayette County High School, Johnny is vice president of his local 4-H Club and has earned more than $2100 through his 4-H projects in the last six years. He looks forward to studying electrical engineering.

Critical steps in the processing of gladioli bulbs are handled by Johnny Clive Williams' ingenious device. Bulbs drop down a vertical chute and at the same time are sprayed with DDT powder to protect them from plant pests. A cotton "poisoner" blows the powder through a vacuum cleaner hose into the chute. Excess powder is sucked into the vacuum bag to be used over again. The bulbs are automatically counted as they drop down the chute, then are tossed onto a sloping conveyor and carried upward onto a receiving screen.

Made of wood, tin, an electric motor, an old vacuum cleaner, a cotton "poisoner," and two conveyor belts, the machine does in 3½ hr. what used to take 24 hr. by hand.

Cement Gun.—The announcement of the invention of a new type of gun for projecting cement, pumice and sand, is attracting the interest of contractors and the building trades throughout the country.

Actually the new Conair gun, which is being manufactured by Conair Sales, Inc. at 12147 Riverside Drive, North Hollywood, California, according to its president, Ludlow Kramer, is a combination of both cement mixer and gun. It is an entirely mobile unit which can move along on every kind of job from house construction to ditch lining without having to have numerous sand or cement piles.

In this Conair gun, invented, perfected, and patented by engineer L. E. Town, and being financed by the Gerry Estates, famous eastern family, there is now incorporated a feed for the material from the machine directly into the air line, which is held at constant pressure, thereby insuring that the sand and cement flow evenly into the hose at all times. This also allows an increase in the amount of material being applied to the job. Also perfected is a system of adding the proper amount of water through a metering device which is coupled with the speed of the flow of the material so that an engineer or inspector can determine exactly the amount of water per cubic yard or per sack of cement in the mix. In so metering the material, the operator of the nozzle knows at all times that he has the proper amount of material flowing through the nozzle and that it is impossible to build an uneven surface. This automatically eliminates the scraping off of excess material or of adding more where there is an insufficient amount.

The simplicity of operation calls for only a crew of nozzle man, a compressor engineer and a rod man.

Other advantages which the Conair gun has over previous and more complicated models are that it has three conventional packing glands on absolutely round shaftings; that the gun is mounted and operates on a roller type bearing throughout. There are no rubber liners in the gun, nor disks or seal rings. The entire gun can be dismantled and reassembled in less than thirty minutes, preventing any escape of air. The pressure guarantee remains constant and shuts off automatically if not kept at the poundage desired.

Since the mounted machine is mobile, only a hundred foot length of hose is required. Also in all operations of Conair gun, the necessity of having large piles of material is eliminated, and there is absolutely no mess or cleaning up process required after the operation is completed.

The new Conair gun is being utilized by Conair Sales Inc. in the manufacture of an all concrete conventional two bedroom home. This home can be erected in twelve hours because of this machinery, at a cost to the public of less than five dollars per square foot. The complete machinery for building these homes can be leased by contractors from Conair Sales Inc., according to Mr. Kramer.

Contax-S Zeiss-Ikon Camera.—A revolutionary new Contax camera from Zeiss-Ikon based on the single-lens reflex principle and called the Contax-S, was introduced to the United States market recently by the Ercona Camera Corporation, sole American sales representatives. Marking the first improvement in Zeiss camera design in 15 years, it incorporates features that greatly expand the scope of the miniature camera without increasing its size. A limited quantlity is now available.

The most radical departure from the previous Contax is the new viewing and focusing system. Here, Zeiss-Ikon has entirely eliminated the familiar twin-image, coupled rangefinder in favor of a single reflex focusing viewer known as the "Prisma-Scope." It enables the photographer to see a life-size, brilliantly sharp image taken directly from the camera lens, while holding the camera at eye level. The Contax-S thus becomes the first miniature camera to use the reflex principle in this manner.

In actual operation, the reflecting mirror throws the image up to a ground glass screen. Here it is picked up and "bent" by a complex prism and lens system so that it reaches the eye of the photographer in a greatly enlarged, upright position. Focusing is accomplished simply by turning the convenient, milled ring of the lens mount. The tremendoulsy magnified, life-size image assures critical focusing to a degree never before possible on a camera of this size.

Other Zeiss-Ikon innovations in the new Contax-S include built-in flash synchronization with the contact ingeniously located inside the tripod socket, allowing for the attachment of a remarkably simple and functional flash gun. All speeds on the focal-plane shutter, from 1 to 1/1000th seconds, are controlled from a single knob located on top of the camera. In addition, no moving parts are exposed, eliminating the danger of the shutter being disturbed while in operation. The shutter release is conveniently located on the camera face to permit quick focusing with the left hand and snapping the picture with the right. A hinged back makes for easy loading with all standard 35 mm. film cartridges.

The only accessory necessary with this new Zeiss-Ikon camera is an adaptor coupling the camera body to the microscope. The object to be photographed may be observed through the regular "Prisma-Scope" finder, which permits continuous viewing of a brilliant image up to the moment of exposure. The penta-roof prism built into the finder insures the image being upright and non-reversed, no matter how the camera is turned. Complete freedom from parallax, high speed and great compactness make the Contax-S an ideal instrument for all phases of scientific photography.

Microfilm Service.—The Franklin Institute has entered into an agreement with University Microfilms, Ann Arbor, Mich., to make available to libraries issues of the JOURNAL OF THE FRANKLIN INSTITUTE in microfilm form.

One of the most pressing problems facing all types of libraries today is that of providing adequate space for a constant flood of publications. Periodicals pose an especially difficult problem because of their bulk and number.

Microfilm makes it possible to produce and distribute copies of periodical literature on the basis of the entire volume in a single roll, in editions of 30 or more, at a cost approximately equal to the cost of binding the same material in a conventional library binding.

Under the plan, the library keeps the printed issues unbound and circulates them in that form for from two to three years, which corresponds to the period of greatest use. When the paper copies begin to wear out or are not called for frequently, they are disposed of and the microfilm is substituted.

Sales are restricted to those subscribing to the paper edition, and the film copy is only distributed at the end of the volume year.

The microfilm is in the form of positive microfilm, and is furnished on metal reels, suitably labeled. Inquiries concerning purchase should be directed to University Microfilms, 313 N. First Street, Ann Arbor, Mich.